Currency Choice in Contracts*

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Abstract

We study the interaction between the currency choice of private domestic contracts and optimal monetary policy. The optimal currency choice depends on the price risk of each currency, as well as on the covariance of its price and the relative consumption needs of the agents signing the contract. When a larger share of contracts is denominated in local currency, the government can use inflation more effectively to either redistribute resources or reduce default costs, which makes local currency more attractive for private contracts. When governments lack commitment, competitive equilibria can be constrained inefficient, thus providing a reason to regulate the currency choice of private contracts. We show that both the equilibrium use of local currency and the implications for regulation depend on the level of domestic policy risk. Our model can explain the wide use of the U.S. dollar in international trade contracts and the observed hysteresis in dollarization.

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1 Introduction

One of the central roles of currency is to serve as a unit of account in private credit contracts. While in most countries this role is exclusively fulfilled by the local currency, several countries also rely on a foreign currency (for example, the U.S. dollar) to denominate domestic contracts. The coexistence of multiple currencies is especially relevant in emerging economies, which are often subject to a greater degree of policy instability. In this paper, we address two related questions on the role of currencies as units of account. First, what determines the currency choice of credit contracts among private agents? Second, how do these individual currency choices affect the government's conduct of monetary policy?

To answer these questions, we study a general equilibrium model in which agents choose the currency in which to denominate contracts, and the government chooses the inflation rate. These contracts involve the provision of a good in exchange for a future payment denominated in some currency. The optimal choice of currency depends on the price risk of each currency, as well as on how this price covaries with the relative consumption needs of the agents signing the contract. The price of the local currency is chosen ex-post by a benevolent government and depends on the use of this currency in private contracts. A key feature of this model is the complementarity between the actions of private agents and those of the government. When a larger share of private contracts is denominated in local currency, the government can use inflation to either redistribute resources more effectively or reduce default costs, which, in turn, makes local currency more attractive as a unit of account for private contracts. The government is also subject to exogenous policy risk, which affects the price risk of local currency and reduces the attractiveness of denominating contracts in local currency. We show that the set of equilibria depends crucially on the level of policy risk, and multiple equilibria can emerge. We also ask whether competitive equilibria are efficient and argue that there might be a role for regulation to encourage private agents to denominate contracts exclusively in one currency. This might help explain policy initiatives in many emerging economies aimed at prohibiting the use of foreign currency in domestic contracts.

At the core of our theory is the debt-deflation channel studied by Fisher (1933) and the ability of governments to use monetary policy as a tool for redistribution in certain states of the world. Indeed, history offers examples of inflation being used to reduce the real value of debt obligations of private agents. A notable example is the experience of the U.S. during the Great Depression, when the continuous decline of commodity prices posed challenges to the highly indebted farm sector. In response to the situation, the Farm Relief Act enacted by Roosevelt paved the way for the abandonment of the gold standard and an increase in inflation. According to Edwards (2018): "This was what the president was after: higher prices that would increase farmers' incomes and would reduce the burden of their debts in real terms."

We begin our analysis by characterizing the optimal bilateral contract. Our framework, which builds on recent work by Doepke and Schneider (2017), nests a variety of contracts in which unit-of-account considerations are present, including debt contracts and trade credit contracts. Buyers and sellers sign contracts to exploit gains from trade of a special good. Contracts stipulate the amount of a special good that is provided at the date the contract is signed, in exchange for an amount of local currency, foreign currency, or both to be paid in the future. Currencies serve only as units of account, since the actual payment in the future is made in terms of a numeraire good. The price levels of both currencies (measured in terms of the numeraire good) are stochastic and unknown at the time contracts are signed. After signing contracts, agents also receive taste shocks, which affect their marginal utility of consuming the numeraire good. This increases the desirability of currencies whose price covaries with these shocks. In addition, agents face a constraint which requires that payments be feasible in all states of the world. This increases the desirability of currencies in which larger payments can be promised without violating this constraint. Consequently, currencies with less extreme price realizations—i.e., lower price risk—are more desirable. The optimal currency choice features a trade-off between these two forces.

In the model, the price of foreign currency is exogenous, while the price of local currency is chosen by a benevolent government that lacks commitment. The government's optimal choice of inflation trades off the benefits of either redistributing resources more effectively or reducing default costs with the costs of deviating from a target. In the baseline model, the benefits of using inflation are to redistribute resources given the differences in the taste shocks of buyers and sellers. We also study a model with costly default in which inflation can help reduce default costs and show that it maps into our baseline setup. The optimal inflation choice redistributes resources between agents in an ex-post efficient way. For example, when buyers have a high marginal utility (relative to sellers), the government chooses higher inflation to lower the real burden of payments. The degree of redistribution that takes place depends positively on the use of local currency in private contracts. The government's inflation choice also depends on the cost of deviating from a target which is stochastic, unknown at the time when contracts are signed, and independent of currency choice. We refer to fluctuations in the target as policy risk.

We fully characterize the set of equilibria for different levels of policy risk. This exercise is motivated by the positive relationship between domestic dollarization and measures of policy risk across countries. One such measure of policy risk is the volatility of government expenditures. As we show in Figure D.1, domestic dollarization is positively correlated with the volatility of government expenditures across countries.[1] For example, the U.S., Germany, and Japan rely exclusively on their respective local currency as a unit of account in domestic contracts, while countries in Latin America and Eastern Europe tend to partially or fully rely on foreign currency as a unit of account. Consistent with this observation, we find that for low levels of policy risk there is a unique equilibrium in which all contracts are denominated in local currency, while for high levels of this risk, all contracts are denominated in foreign currency. For intermediate levels of policy risk there are three equilibria: two of which involve the exclusive use of either local or foreign currency, and a third interior one in which both local and foreign currencies are used. We then use a global games refinement to uniquely select an equilibrium for any level of policy risk and find that there is a unique cutoff below which all contracts are denominated in local currency and above which all contracts are denominated in foreign currency.

Both recently and historically, many countries have introduced policy initiatives which either encourage or discourage the use of foreign currency as a unit of account. On the one hand, there have been policy initiatives in a large number of emerging market economies that discourage the use of foreign currency as a unit of account. Two such examples are Brazil and Colombia, which prohibit the denomination of bank deposit and loan contracts in foreign currency. Other similar examples of recent initiatives include policies in Hungary and Poland, which either heavily regulated or forced the conversion of foreign currency housing loans to domestic currency. On the other hand, two decades ago Ecuador and El Salvador fully dollarized their domestic economies.

Our paper can help rationalize the prevalence of such policy initiatives. We study the problem of a social planner subject to the same constraints as private agents. We find that the optimal allocation is characterized by a cutoff in policy risk below which all contracts are denominated in local currency and above which all contracts are denominated in foreign currency. Additionally, in the region of policy risk with multiple equilibria, we find that for low levels of policy risk, equilibria with foreign currency use are dominated by one with full use of local currency, and for high levels of policy risk, equilibria with local currency use are dominated by one with full use of foreign currency.

There are two sources of inefficiency in private currency choices stemming from the fact that, being of measure zero, individual agents do not internalize the effect of their actions on the policy choice of the government. The first arises as a consequence of complementarities between the private and government actions, which imply that the private marginal benefit of denominating contracts in local currency is increasing in the aggregate

[1] Other measures of policy risk, based on institutional factors, have been shown to covary positively with dollarization (Nicolo et al. (2003b), Rennhack and Nozaki (2006)). Note that we focus on one measure of domestic dollarization of contracts, namely, the share of dollar-denominated bank deposit contracts. Similar patterns are observed if we focus on other measures, including the share of dollar-denominated bank loan contracts.

stock of local currency contracts. Second, private agents do not internalize the inflation costs associated with deviating from the target. While the first source leads to less use of local currency than is optimal, the second source has the opposite effect. To characterize the optimal regulation as a function of parameters, we then compare the planner's cutoff of policy risk to the unique cutoff selected by the global games refinement. We find that, depending on parameters, the planner's cutoff can be strictly lower or higher than the global games cutoff. In particular, there is a region of inefficiency in which the unique competitive equilibrium calls for less local currency use than the planner's solution, and vice versa.

We then use our model to study a variety of applications and extensions. First, we study a model with default in which the role of policy is to reduce the costs associated with default. We show that there exist processes for the taste shocks so that the equilibrium outcomes in the baseline model with taste shocks are identical to those in the model with default. Thus, we can apply the results from the baseline to the default model. Moreover, this shows that the taste shock model is quite general and can be used to study other interesting environments. Another takeaway from this application is that the observed inflation policy need not always reflect a redistributive motive. Indeed, during normal times, when there is no risk of default, inflation is set at its target. However, during crises, when default imposes large social costs, the government chooses inflation to reduce the burden of default and redistribute resources.

Second, we extend our model to study currency choice in international trade contracts. Previous literature (see Goldberg and Tille (2009); Goldberg (2013); Ito and Chinn (2013); Gopinath (2016)) has documented that the U.S. dollar is widely used as a unit of account in international trade contracts. In particular, countries such as Japan have low inflation risk and low domestic dollarization, and yet have a significant fraction of their international trade contracts denominated in dollars. Moreover, as we document in Figure D.2, the share of import contracts denominated in U.S. dollars exceeds the share of dollar-denominated domestic financial contracts in most countries. This suggests that the use of the dollar is more prevalent in international contracts than in domestic ones. Motivated by this observation, we study a two-country model where, in addition to domestic contracts, there are international contracts in which buyers (respectively, sellers) in one country trade with sellers (respectively, buyers) from another symmetric country, and contracts can be set in three possible currencies: the currencies of either country and a foreign currency (which in this case stands for the U.S. dollar). Our model can rationalize the larger use of the dollar in international contracts relative to domestic contracts. We show that there exist levels of policy risk such that a full local currency equilibrium exists for domestic contracts, but not for international contracts. In particular, in this range of policy risk agents strictly prefer to denominate international contracts in foreign currency (the dollar), while they prefer to denominate domestic contracts in local currency if all other agents do so. The reason is that the benefit for an agent to denominate contracts in the local currency of its trading partner is lower if the partner is from a different country. This is because the government has incentives to respond only to the taste shocks of its own citizens and not to those of other countries' citizens. In contrast, for domestic contracts, the government responds to the taste shocks of both partners involved, thus raising the insurance benefit of denominating contracts in the local currency.

Finally, we use our model to shed light on the observed hysteresis in the share of contracts denominated in foreign currency. This pattern is most striking in many Latin American economies that still exhibit high levels of financial dollarization in spite of continued success in controlling inflation and inflation risk in the last decade (Ize and Levy-Yeyati (2003)). To address this empirical pattern, we enhance our baseline model by endowing buyers with claims on local and foreign currency that, as we show, can arise endogenously as a consequence of trading within a credit chain. In this model, currency choice exhibits hysteresis because there are benefits of matching the currency of denomination of new contracts to the currency of outstanding claims of buyers, as doing so reduces the exposure to price risk. We illustrate this by showing that even if policy risk gets arbitrarily small, in equilibrium, foreign currency will still be used as a unit of account. The reason is that it is optimal to match the currency of the stock of existing claims held by buyers and only de-dollarize the flow of additional payments.

Related Literature There is a large literature that studies the use of currencies for a variety of purposes. Our paper is related to a literature that studies the choice of currency denomination of debt contracts (see Ize and Levy-Yeyati (2003), Caballero and Krishnamurthy (2003), Schneider and Tornell (2004), Doepke and Schneider (2017), and Bocola and Lorenzoni (2019), among others).[2] These papers abstract from the interaction between private currency choices and monetary policy, which is the focus of our paper. Our contracting framework builds on Doepke and Schneider (2017), who study the determination of a unit of account in the presence of exogenous price risk. Our environment is different in two key ways. First, our framework features a trade-off between price risk and the insurance properties of each currency, which is absent in their paper. Second, and more importantly, we model the optimal conduct of monetary policy, thus endogenizing both the price risk and insurance benefits, and focus on the interaction between private choices of the unit of account and the government's policy choices. These two

[2]Currency choice has also been studied in the context of denomination of prices (see, for example, Devereux and Engel (2003), Bacchetta and Van Wincoop (2005), Engel (2006), Goldberg and Tille (2008), Gopinath et al. (2010), Corsetti et al. (2015), Gopinath et al. (2018), and Drenik and Perez (2019)), and means of payment (see Matsuyama et al. (1993) and Uribe (1997)).

differences are fundamental to the characterization of equilibria and the implications for optimal regulation of currency choices.

Our paper contributes to the literature that studies the interaction between currency choice and policy. First, there are papers in which both the currency and policy choices are made by governments. Ottonello and Perez (2019), Du et al. (2019), and Engel and Park (2019) study the interaction between monetary policy and the currency denomination of sovereign debt. Neumeyer (1998), Alesina and Barro (2002), Arellano and Heathcote (2010), and Chari et al. (2019) study the trade-offs associated with forming currency unions or dollarizing the economy. In contrast to these papers, our paper focuses on private agents' currency choices and how they interact with those of the government.

Second, there is a set of papers that study the interaction between the currency choices of private agents and monetary policy. Svensson (1989), Chang and Velasco (2006), and Devereux and Sutherland (2008) analyze the optimal portfolio choice when there are nominal assets, for different monetary policy rules. Rappoport (2009) studies a model of currency choice in corporate debt to rationalize the prevalence of hysteresis in domestic dollarization. Fanelli (2019) studies the interaction between private debt choices and exchange rate policies when governments can commit. We contribute to this literature in two key dimensions. First, we study a model in which governments choose monetary policy without commitment. The lack of commitment can give rise to equilibrium multiplicity and inefficiency. Crucially, we show that both the equilibrium set and the existence and type of inefficiencies depend on the level of policy risk. In this sense, our results can rationalize the cross-country heterogeneity in the use of the dollar in domestic contracts, and shed light on the current debates surrounding the regulation of domestic dollarization in various countries. Second, our general framework allows us to study a variety of applications including the role of inflation to mitigate default costs, international trade contracts, and hysteresis in dollarization.

Finally, our paper contributes to a growing literature on the global role of the dollar (see, for example, Maggiori (2017), Farhi and Maggiori (2017), Gopinath and Stein (2019), Chahrour and Valchev (2019), Maggiori et al. (2019), Mukhin (2019), and Eren and Malamud (2019)). Gopinath and Stein (2019) emphasize a complementarity between the use of the dollar for invoicing in international trade and the aggregate demand for dollar-safe assets. We propose a complementary view to theirs, which relies on the interaction between private currency choices and governments' policy choices. Our theory can help account for the relatively high use of the dollar in countries with greater policy risk, as well as the greater use of this currency in international contracts relative to domestic ones.

The rest of the paper is organized as follows. Section 2 presents the model, characterizes the equilibrium, and analyzes the constrained efficient allocation of the economy. In section 3, we study a variety of applications of our baseline model, including strategic default (subsection 3.1) and international trade contracts (subsection 3.2), and analyze the observed hysteresis in the currency of contracts (subsection 3.3). We present our conclusions in section 4.

2 Model

In this section, we develop a model to study the interaction between the currency choice of private contracts and optimal monetary policy. Our model is flexible enough to incorporate a variety of settings in which currency choice is important, for example, trade-credit and debt contracts.

First, we describe the competitive equilibrium keeping the government's policies fixed in order to highlight the trade-offs private agents face when choosing the currency of denomination of contracts. In the following subsections, we characterize the full equilibrium with endogenous government policy and compare it with the efficient allocation.

2.1 General Environment

There are two periods, $t = 1, 2$. The domestic economy is populated by two types of agents: citizens and a government. Citizens are further divided into sellers and buyers, with a unit measure of each.

Buyers have preferences over consumption of a special good produced by sellers in period 1. Buyers and sellers also value the consumption of a numeraire good, which takes place in period 2. The preferences of the representative seller are given by

$$u_s = -x + \mathbb{E}[\theta_s c_s],$$

where x is the special good produced by the seller, c_s is the seller's consumption of the numeraire good, and θ_s is a taste shock which measures the seller's marginal utility of consuming the numeraire good. The preferences of the representative buyer are given by

$$u_b = (1+\lambda)x + \mathbb{E}[\theta_b c_b],$$

where $1+\lambda$ is the valuation of the special good provided by the seller, c_b is the buyer's consumption of the numeraire good, and θ_b is the buyer's taste shock.[3] The parameter $\lambda \geqslant 0$ governs the gains of trading the special good between sellers and buyers. We assume that θ_s is drawn from a distribution with mean $\mathbb{E}[\theta_s]$ and support $[\underline{\theta}_s, \overline{\theta}_s]$ and that

[3]Note that θ_s and θ_b are shocks to the *representative* buyers and sellers, respectively. Since preferences are linear, and there is a continuum of agents, these shocks correspond to the aggregate component of individual shocks.

θ_b is drawn from a distribution with mean $\mathbb{E}[\theta_b]$ and support $[\underline{\theta}_b, \overline{\theta}_b]$. We make no assumption about the correlation between θ_s and θ_b. The fact that θ_s and θ_b are unknown in period 1 introduces uncertainty in the relative marginal utilities of the numeraire good and gives rise to gains from making relative consumption state-contingent. A high (respectively, low) value of θ_b relative to θ_s makes the consumption of buyers, relative to sellers, more (respectively, less) desirable. As we will see, these taste shocks are a stylized way of generating the value of having a flexible government policy. The differences in θ_s and θ_b can capture any reason it is socially and privately desirable to shift resources between different groups of citizens in the population. One such reason could be the desire to reduce the incidence and burden of default for certain agents. For example, as we show in Section 3.1, a model with default and stochastic default costs maps directly into our environment. In that model, the taste shocks correspond to the default costs faced by buyers. Finally, buyers and sellers are endowed with $y > 0$ units of the numeraire good in period 2.

The timing of the model is as follows:

1. In period 1, sellers produce a special good for buyers in exchange for the promise of a payment in period 2.

2. In period 2, taste shocks θ_s and θ_b are realized, the domestic government chooses its policy consisting of the aggregate price level, all signed contracts are executed, and consumption of the numeraire good takes place.

Next, we formally define a contract and discuss its properties.

2.2 Bilateral Contracts

A contract between a buyer and a seller consists of the provision of the special good (from the seller to the buyer) in exchange for the promise of future payment (from the buyer to the seller). We impose three important assumptions on the contracting environment. The first is that payments are non-contingent and, in particular, cannot depend on the realization of the state (θ_s, θ_b). The second is that payments can be denominated in two possible "units of account", which we call *currencies*. We denote the two possible currencies by l (local) and f (foreign), which can represent, for example, "pesos" and "dollars", respectively. A payment b_l in currency l yields $b_l\phi_l$ units of the domestic numeraire good in period 2, while a payment b_f in currency f yields $b_f\phi_f$ units of the domestic numeraire good in period 2. Here, ϕ_l and ϕ_f denote the price of the local and foreign currencies in terms of the numeraire good, respectively. In general, ϕ_l and ϕ_f are random variables from the perspective of private agents that are unknown at the time the contract is signed. The third assumption is that default costs are sufficiently high so that contracts must be *default-free*. In other words, actual payments must equal promised payments in all states of the world. We relax this assumption in Section 3.1.

Formally, a bilateral signed contract is a tuple (x, b_l, b_f), where x indicates the units of the special good provided to the buyer, and (b_l, b_f) are the units of local and foreign currency promised to be paid to the seller at date 2, respectively. The assumption that contracts must be default-free, along with a non-negativity constraint on the buyer's consumption, implies that contracts must satisfy the following payments feasibility constraint:

$$b_l\phi_l + b_f\phi_f \leqslant y \ \forall \left(\phi_l, \phi_f\right) \in \Phi, \tag{1}$$

where $\Phi = \left[\underline{\phi}_l, \overline{\phi}_l\right] \times \left[\underline{\phi}_f, \overline{\phi}_f\right]$ is the set of all possible price realizations. This inequality states that for all possible price realizations, the promised repayment must not exceed the income of the buyer. Citizens take prices ϕ_l and ϕ_f as given and are exposed to risk from uncertainty about these prices. We conjecture that ϕ_l and ϕ_f are independent random variables from the perspective of private agents. In the next subsection, we model ϕ_l as being chosen by a benevolent government. Thus, the distribution and support of ϕ_l are determined in equilibrium, and we verify that it is indeed independent of ϕ_f. Consequently, this conjecture is without loss of generality. The foreign currency price ϕ_f is assumed to be exogenous and independent of all other random variables (including θ_s and θ_b). We associate the foreign currency with relatively stable currencies, such as the U.S. dollar or the euro, and interpret the risk in ϕ_f as real exchange rate risk.[4] Note that a low (respectively, high) value of ϕ_c indicates a high (respectively, low) level of domestic inflation in currency c. Throughout the paper, we refer to "inflation" and "price level" interchangeably.

We assume that in each bilateral meeting the buyer and the seller choose a contract that maximizes the sum of their utilities. The seller is willing to participate in the contract as long as

$$-x + \mathbb{E}\left[\theta_s\left(y + b_l\phi_l + b_f\phi_f\right)\right] \geqslant \mathbb{E}\left[\theta_s y\right], \tag{2}$$

where the outside option of the seller is to consume their endowment. Similarly, the buyer

[4] In Appendix C.1, we show how risk in ϕ_f can arise in a model with tradable and non-tradable goods and shocks to the relative demand of these goods. It is also worth noting that while we do not explicitly allow for hedging against foreign currency price movements, this is implicitly captured by the properties of the distribution of ϕ_f. We make no assumptions about this distribution. In particular, the case in which ϕ_f is deterministic can be interpreted as a situation in which private agents can completely insure the risks of denominating contracts in foreign currency, or, alternatively, contracts are denominated in the numeraire good.

is willing to participate in the contract as long as

$$(1+\lambda)\,x+\mathbb{E}\left[\theta_b\left(y-b_l\phi_l-b_f\phi_f\right)\right]\geqslant\mathbb{E}\left[\theta_b y\right]. \tag{3}$$

Thus, the privately optimal contract solves

$$\max_{x,b_l,b_f}\,(1+\lambda)\,x-\mathbb{E}\left[\theta_b\left(b_l\phi_l+b_f\phi_f\right)\right]-x+\mathbb{E}\left[\theta_s\left(b_l\phi_l+b_f\phi_f\right)\right] \tag{4}$$

subject to (1), (2), (3), and the non-negativity constraints $b_l, b_f \geqslant 0$.

There are two points worth noting about the contracting problem. First, the choice of the objective (4) is not restrictive. In fact, the optimal contract coincides with one in which the buyer makes a take-it-or-leave-it offer. Second, we impose the non-negativity constraints $b_l, b_f \geqslant 0$ because these correspond to payments made by the buyer in exchange for the special good x.[5] Moreover, in Section 3.3, we show that under a tighter parametric condition, even if we allow buyers to promise negative payments (i.e., payments from the seller to the buyer) in a certain currency, these will not be part of the optimal contract.

In order to characterize the solution to problem (4), we make the following assumption guaranteeing that buyers and sellers find it worthwhile to sign a contract in which $x > 0$.

Assumption 1. *Assume that*

$$(1+\lambda)\,\mathbb{E}\left[\theta_s\right]-\mathbb{E}\left[\theta_b\right]>0.$$

It is worth noting that our setup nests two types of contracts in which currency choice is important. The first is a trade-credit contract, in which gains from trade arise from the static exchange of the special good in period 1. This corresponds to the case in which $\mathbb{E}[\theta_b] = \mathbb{E}[\theta_s]$ and $\lambda > 0$. The second is a standard debt contract, in which gains from trade arise from the intertemporal exchange of goods. In particular, one can interpret differences in expected taste shocks between buyers and sellers as heterogeneity in discount factors. Therefore, if $\mathbb{E}[\theta_b] < \mathbb{E}[\theta_s]$ and $\lambda = 0$, there are no static gains from trading but the buyer is relatively more impatient than the seller and thus would like to borrow in period 1. Under this interpretation, x corresponds to the amount borrowed by the buyer. Consequently, the labels of "special" and "numeraire" goods are merely used to distinguish goods traded in periods 1 and 2, respectively. Similarly, the labels of "buyers" and

[5] Since the seller provides the good to the buyer, at least one of the payments b_l or b_f must be positive. Thus, what we are ruling out are contracts in which the buyer makes a positive payment to the seller in one currency and the seller makes a positive payment to the buyer in the other currency, within the same contract. While such contracts might exist in theory, we think that they are empirically less relevant.

"sellers" are interchangeable with "borrowers" and "lenders", respectively.[6]

While in this section we study a bilateral contracting problem, in Appendix C.2 we show that the equilibrium allocations with such contracts are identical to those in an economy with centralized markets with local and foreign currency debt. There, we also provide a discussion about the equilibrium interest rates and show how they relate to the objects in our economy.

2.3 Competitive Equilibrium Given Government Policy

We now characterize the optimal bilateral contract between a seller and a buyer, taking the distribution of ϕ_l and ϕ_f as given. Since preferences are linear and $\lambda \geqslant 0$, there are positive gains from trading as much of the special good x as possible. This implies that the seller's participation constraint will bind so that the amount of the special good is determined by the value of promised payments made to the seller.[7] In turn, these promised payments are limited by the fact that buyers need to be able to pay for that good in the following period, i.e., by the payments feasibility constraint (1). Assumption 1 implies that the payments feasibility constraint will always be binding. Since agents perceive ϕ_l and ϕ_f as independent random variables, the state for which this constraint will bind is the one in which inflation $1/\phi_c$ in both currencies is at its lowest possible realization (i.e., $\phi_l = \overline{\phi}_l$ and $\phi_f = \overline{\phi}_f$). If we substitute the binding participation constraint of the seller and the feasibility constraint into the objective, the derivative with respect to b_l is proportional to

$$\underbrace{\mathbb{E}\left[(\theta_s(1+\lambda)-\theta_b)\frac{\phi_l}{\overline{\phi}_l}\right]}_{\text{Marginal benefit of local currency }(M_l)} - \underbrace{\mathbb{E}\left[(\theta_s(1+\lambda)-\theta_b)\frac{\phi_f}{\overline{\phi}_f}\right]}_{\text{Marginal benefit of foreign currency }(M_f)}.$$

The expression above represents the difference between the marginal benefit of setting the contract in the local currency (M_l) and the marginal benefit of setting it in the foreign currency (M_f). Since the objective is linear, these objects are constant and independent of the choice of b_l. The optimal contract calls for using the currency that has the largest marginal benefit. When the marginal benefit is the same in both currencies, any combination of local and foreign currency is optimal. The following proposition formalizes this result.

[6]These broad classes of agents can have different interpretations depending on the particular application. For example, in the context of the U.S. Great Depression discussed in the introduction, "buyers" would refer to the farmers who required debt to finance production, and "sellers" to their creditors. In other relevant applications, "buyers" would refer to firms taking on debt, or banks taking deposits, and "sellers" would refer to households. In the case of international trade contracts, analyzed in Section 3.2, "buyers" would refer to importers that make purchases with trade credit from exporters ("sellers").

[7]We show in the proof of Proposition 1 that the participation constraint for the buyer is always slack.

Proposition 1. *Suppose that Assumption 1 holds. In the optimal bilateral contract, the amount of special good is given by* $x = \mathbb{E}\left[\theta_s\left(b_l\phi_l + b_f\phi_f\right)\right]$, *while the payments satisfy*

1. *If* $M_l < M_f$, *then* $b_l = 0$ *and* $b_f = \frac{y}{\overline{\phi}_f}$
2. *If* $M_l = M_f$, *then* $b_l = \rho\frac{y}{\overline{\phi}_l}$ *and* $b_f = (1-\rho)\frac{y}{\overline{\phi}_f}$ *for any* $\rho \in [0,1]$.
3. *If* $M_l > M_f$, *then* $b_l = \frac{y}{\overline{\phi}_l}$ *and* $b_f = 0$.

All proofs are included in the Appendix. To understand the marginal benefit of denominating the contract in a currency c, we can rewrite it as

$$M_c \equiv \left[(1+\lambda)\,\mathbb{E}\left[\theta_s\right] - \mathbb{E}\left[\theta_b\right]\right]\frac{\mathbb{E}\left[\phi_c\right]}{\overline{\phi}_c} + \text{cov}\left(\left(\theta_s\left(1+\lambda\right)-\theta_b\right), \frac{\phi_c}{\overline{\phi}_c}\right) \tag{5}$$

for $c = l, f$. The marginal benefit of each currency has two components: a price risk term and a covariance term. The ratio $\mathbb{E}\left[\phi_c\right]/\overline{\phi}_c$ denotes the price risk of denominating contracts in currency c. A higher (respectively, lower) value of $\mathbb{E}\left[\phi_c\right]/\overline{\phi}_c$ represents a lower (respectively, higher) risk of indexing contracts in currency c. Note that it is the maximal value $\overline{\phi}_c$ that determines price risk due to the assumption that payments must be feasible in all states of the world, in particular in the state with the highest value of currency c in terms of the numeraire good. The second term is the covariance of relative taste shocks and currency prices. The marginal benefit of denominating the contract in foreign currency is exogenous and given only by the price risk term, since the covariance term is zero given our assumption of independence between ϕ_f and the shocks θ_b and θ_s.

To understand the results in Proposition 1, suppose first that θ_b and θ_s are deterministic. Then, the optimal currency choice is determined exclusively by comparing the price risk in both currencies, $\mathbb{E}\left[\phi_l\right]/\overline{\phi}_l - \mathbb{E}\left[\phi_f\right]/\overline{\phi}_f$. In this case, choosing the currency with the lowest price risk maximizes the gains from trade, as it allows buyers to promise sellers larger payments in period 2. In contrast, suppose that the taste parameters are stochastic. Now the optimal currency choice also depends on the covariance between prices in local currency and marginal utilities (taste shocks). For example, if ϕ_l is high in the states in which the seller values consumption relatively more than the buyer does (high θ_s relative to θ_b), denominating the contract in local currency is more attractive. As we will see in the next section, a benevolent government will choose ϕ_l so that this covariance term is positive. Finally, the optimal choice of x can be computed directly from the participation constraint of the seller (2).

At this point, it is worth describing the differences between our results and those in Doepke and Schneider (2017), who also study the determination of the optimal unit of account. First, Doepke and Schneider (2017) only focus on differences in price risk. Therefore, the trade-off between relative price risk and covariance benefits, characterized in our

Proposition 1, is absent in their paper. Second, as we describe in the next section, in our paper, local currency prices are determined by a government, which in turn generates complementarities between private and government actions. These two differences are fundamental to the characterization of equilibria and the implications for optimal regulation of private currency choices.

2.4 Government

We consider a utilitarian government that controls monetary policy and chooses the price level of the domestic economy ϕ_l in the second period to maximize the sum of the utilities of buyers and sellers net of the losses associated with inflation, captured by $l(\phi_l)$. We assume that $l(\phi_l) = \frac{\psi}{2}\left(\phi_l - \hat{\phi}\right)^2$, where $\hat{\phi}$ denotes the price level targeted by the government in the absence of redistributional concerns. The target $\hat{\phi}$ is a random variable realized in period 2 and, thus, stochastic at the time contracts are signed. We assume that $\hat{\phi}$ is independent of ϕ_f, θ_s, and θ_b, and has bounded support $\left[\underline{\hat{\phi}}, \overline{\hat{\phi}}\right]$. Similar to our definition of price risk, we refer to $\mathbb{E}\left[\hat{\phi}\right]/\overline{\hat{\phi}}$ as policy risk. As before, a higher (respectively, lower) value of $\mathbb{E}\left[\hat{\phi}\right]/\overline{\hat{\phi}}$ represents a lower (respectively, higher) policy risk. The target $\hat{\phi}$ captures other reasons that determine the optimal price level. In Appendix C.3, we provide a microfoundation of the inflation loss function from the Ramsey problem of a government facing spending shocks that raises revenue through a combination of distortionary taxation and seigniorage. The target $\hat{\phi}$ denotes the optimal level of inflation for a given spending level. Thus, the loss function captures the costs of deviating from this optimal policy.[8] Other determinants of $\hat{\phi}$ could include the stabilization of output or the price level.

An important assumption implied by the timing above is that the government lacks commitment. This choice is motivated by the fact that in reality governments find it hard to commit to state contingent policies. This is particularly true in emerging economies which tend to display higher levels of domestic dollarization. In Appendix C.5, we describe the problem with commitment. We show that in this case, the equilibrium is efficient. As we will see, this is in sharp contrast to equilibria without commitment.

Without commitment, the problem of the government is given by

[8] In this case, the use of inflation to collect seigniorage relies on the use of local currency as a means of payment, but not on the aggregate promised payments denominated in the local currency, B_l. One can think of other channels through which the use of local currency in domestic contracts may affect the losses associated with inflation (for example, if the use of local currency as unit of account in credit contracts is complementary to its use as means of payments). Our model can incorporate such cases if, for example, the loss function takes the form $l(\phi_l) = \frac{(\psi + f(B_l))}{2}\left(\phi_l - \hat{\phi}\right)^2$, given some function $f(B_l)$. As long as $f'(B_l)$ is not too large, one can show that the main trade-offs that characterize the set of competitive equilibria in the baseline are still present in this model.

$$\max_{\phi_l} \left[\theta_b C_b + \theta_s C_s\right] - l\left(\phi_l\right),$$

where

$$C_b = y - \phi_l B_l - \phi_f B_f \tag{6}$$

is the aggregate consumption of buyers, B_l and B_f are the aggregate promised payments denominated in the local and foreign currency, respectively, and

$$C_s = y + \phi_l B_l + \phi_f B_f \tag{7}$$

is the aggregate consumption of sellers.[9]

Given the functional form of $l(\cdot)$, the solution to the government's problem is

$$\phi_l = \hat{\phi} + \frac{1}{\psi}\left(\theta_s - \theta_b\right) B_l. \tag{8}$$

The optimal choice of inflation redistributes resources between sellers and buyers in an ex-post efficient way.[10] When buyers have a high marginal utility (relative to sellers), the government chooses a higher inflation (lower ϕ_l) to lower the burden of debt payments by the buyer and redistribute resources from sellers to buyers. The opposite occurs when sellers have a high marginal utility relative to buyers. In this model, the choice of monetary policy is governed by redistributional concerns. In Section 3.1, we study a model with costly default in which the role of monetary policy is to reduce default costs, and show that such a model maps directly into this baseline environment.

The government's choice of inflation affects the marginal benefit of setting contracts in the local currency, M_l (defined in equation (5)) in the first period. On the one hand, the redistribution that the government attains using monetary policy induces a positive covariance between relative marginal utilities and the price of the local currency, thereby providing more insurance and increasing the marginal benefit of this currency. The higher the use of the local currency, B_l, the higher the endogenous positive covariance for this currency. In this sense, the government's conduct of monetary policy helps make nominal contracts state-contingent in a desirable way. On the other hand, by reacting to taste shocks, the government also affects the price risk of the local currency. Recall that we

[9]Recall that we imposed a non-negativity constraint on the buyer's consumption in the contracting problem, which is not imposed in the government's problem. This is not a concern since this constraint will never be violated in equilibrium, as private agents will always choose contracts that respect it. However, including this constraint in the government's problem can give rise to additional peculiar equilibria in which the government's choice of inflation is driven purely by the need to satisfy the non-negativity constraint of private agents. We abstract from such equilibria.

[10]Notice that the price level, ϕ_l, is independent of ϕ_f, which verifies the conjecture made in Section 2.2. Additionally, the price level is always positive if $\underline{\hat{\phi}}$ is large enough.

defined the price risk of the local currency as the ratio $\mathbb{E}[\phi_l]/\overline{\phi}_l$. Given the optimal choice of ϕ_l, we have that $\mathbb{E}[\phi_l] = \mathbb{E}\left[\hat{\phi}\right] + \frac{1}{\psi}(\mathbb{E}[\theta_s] - \mathbb{E}[\theta_b]) B_l$ and the maximal value of ϕ_l is given by

$$\overline{\phi}_l = \overline{\hat{\phi}} + \frac{1}{\psi}\left(\overline{\theta}_s - \underline{\theta}_b\right) B_l. \tag{9}$$

The higher the use of the local currency, B_l, the higher $\overline{\phi}_l$, which in turn can lead to a lower $\mathbb{E}[\phi_l]/\overline{\phi}_l$ (or a higher price risk of the local currency). Throughout our baseline analysis we make the following parametric assumption guaranteeing that the net insurance benefit of the local currency (i.e., the increase in covariance net of price risk from higher B_l) is large enough.

Assumption 2. *Assume that*

$$\frac{1}{2} var(\theta_s - \theta_b) + \lambda [var(\theta_s) - cov(\theta_s, \theta_b)] \geqslant \kappa_1$$

where κ_1 is a constant depending on the model parameters defined in (18) in the Appendix.

As mentioned previously, introducing taste shocks is a simple way of generating value for flexibility in monetary policy. Thus, the variance of the relative taste shocks captures the importance of flexibility. Assumption 2 ensures that the value of flexibility is sufficiently large. To understand this assumption, it is instructive to consider the case in which θ_s and θ_b are independent and identically distributed with $\text{var}(\theta_s) = \text{var}(\theta_b) = \text{var}(\theta)$ and $\mathbb{E}[\theta_s] = \mathbb{E}[\theta_b] = \mathbb{E}[\theta]$. Then, this assumption reduces to

$$\text{var}(\theta) \geqslant \frac{\lambda}{(1+\lambda)} \mathbb{E}[\theta] \left(\frac{\mathbb{E}[\phi_f]}{\overline{\phi}_f} \left(\overline{\theta} - \underline{\theta}\right) \right), \tag{10}$$

which is satisfied if the variance is large enough. If Assumption 2 is violated, the covariance benefits arising from denominating contracts in local currency are relatively small. As a result, currency choices in contracts are primarily governed by price risk. We characterize equilibria when this assumption is violated in Appendix C.4. However, we consider Assumption 2 to be empirically relevant, as we want to consider economies in which there arguably is a sizable need and benefit of insurance because private markets are not sufficiently developed. This is typically the case in emerging economies in which currency choice is particularly relevant.

Let$M_l(B_l)$ denote the marginal benefit of denominating contracts in the local currency (defined in (5)), once we substitute in the optimal choice of ϕ_l made by the government. Assumption 2 also guarantees that $M_l(B_l)$ is increasing in B_l. In particular, it guarantees that the positive effect of higher B_l on the covariance term more than offsets the effect of higher B_l on the price risk of the local currency. Therefore, under this assumption, the benefit of denominating contracts in the local currency is increasing in B_l, thus generating complementarities in currency choices.

Given this, we can now define a competitive equilibrium for this economy.

Definition 1. *A competitive equilibrium is an allocation for private citizens* (x, b_l, b_f), *aggregate promised payments* (B_l, B_f), *and an inflation choice of the government* ϕ_l *such that: 1. Given* ϕ_l, *the private allocation solves the contracting problem defined in (4), 2. Given* B_l, ϕ_l *satisfies (8), and 3. Aggregate choices coincide with private ones,* $b_l = B_l$ *and* $b_f = B_f$.

2.5 Equilibrium Characterization

We now provide a characterization of the set of competitive equilibria. As mentioned in the introduction, there is substantial heterogeneity across countries in the use of foreign currencies as units of account in domestic contracts. The goal of this exercise is to understand the factors which drive this heterogeneity. In particular, the main proposition of this section describes how the set of equilibria changes as we vary the level of policy risk. As we will show, for low levels of this risk, there is a unique equilibrium in which all contracts are denominated in local currency. For intermediate levels of this risk, there are three equilibria: two in which all contracts are completely denominated in either local or foreign currency, and an interior equilibrium. Finally, for high enough levels of policy risk, there is a unique equilibrium in which all contracts are denominated in foreign currency. In the next subsection, we will use a global games refinement to select a unique equilibrium given a level of policy risk.

To vary policy risk, we fix $\overline{\hat{\phi}}$ and vary $\mathbb{E}\left[\hat{\phi}\right]$. In particular, a higher value of $\mathbb{E}\left[\hat{\phi}\right]$ denotes a lower level of policy risk. The set of equilibria is characterized in the following proposition.

Proposition 2. *Suppose that Assumptions 1 and 2 hold. Then, there exist thresholds* $\mu_1 = \mathbb{E}\left[\phi_f\right]/\overline{\phi}_f$ *and* $\mu_2 < \mu_1$ *such that:*

1. *If* $\mathbb{E}\left[\hat{\phi}\right]/\overline{\hat{\phi}} > \mu_1$, *there exists a unique equilibrium in which* $B_l = y/\overline{\phi}_l^*$, *where* $\overline{\phi}_l^*$ *is the positive solution to*
$$\overline{\phi}_l^* = \overline{\hat{\phi}} + \frac{1}{\psi}\left(\overline{\theta}_s - \underline{\theta}_b\right)\frac{y}{\overline{\phi}_l^*}.$$

2. *If* $\mu_2 < \mathbb{E}\left[\hat{\phi}\right]/\overline{\hat{\phi}} \leqslant \mu_1$, *there exist three equilibria:* $B_l = y/\overline{\phi}_l^*$, $B_l = 0$, *and* $B_l \in \left(0, y/\overline{\phi}_l^*\right)$.

3. *If* $\mathbb{E}\left[\hat{\phi}\right]/\overline{\hat{\phi}} \leqslant \mu_2$, *there exists a unique equilibrium in which* $B_l = 0$.

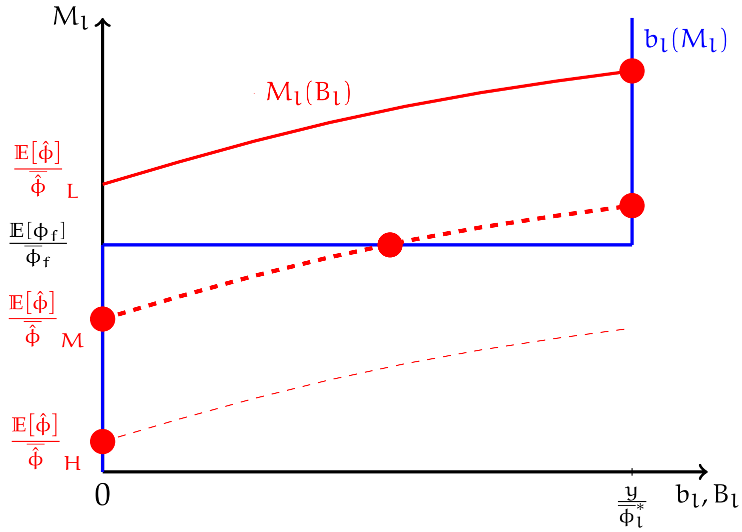

Figure 1: Characterization of competitive equilibrium

The threshold μ_2 depends on parameters and is defined in equation (21) in the Appendix. Figure 1 presents a graphical depiction of the set of equilibria when $\mathbb{E}[\theta_s] = \mathbb{E}[\theta_b] = 1$ and $\lambda = 1$. The blue line reproduces the result in Proposition 1, by depicting the individual optimal promised payment denominated in local currency b_l for a given M_l (the function denoted by $b_l(M_l)$). When $M_l > M_f = \mathbb{E}[\phi_f]/\overline{\phi}_f$, private agents denominate contracts in the local currency, and when $M_l < M_f$ they denominate them in foreign currency. The red lines depict the marginal benefit of the local currency as a function of B_l—i.e., once we substitute in the optimal inflation choice by the government—(the function denoted by $M_l(B_l)$). We plot three lines corresponding to different levels of policy risk. All red lines are increasing since Assumption 2 implies that $M_l(B_l)$ is increasing. Finally, notice that the equilibrium is obtained by computing the fixed point of these two functions evaluated at $b_l = B_l$ (i.e., where the blue and red lines intersect).

To understand the role of policy risk in the determination of equilibria it is useful to analyze how policy risk affects the marginal benefit of local currency. Note that when there are no contracts in the local currency, the optimal inflation choice is equal to the target, and the marginal benefit of the local currency is determined only by policy risk, i.e., $M_l(0) = \mathbb{E}\left[\hat{\phi}\right]/\overline{\hat{\phi}}$. As we increase policy risk (decrease the ratio), the marginal benefit of the local currency decreases for all possible values of B_l. When policy risk is lower than the price risk of foreign currency (case 1), the unique equilibrium uses only local currency, as shown at the intersection of the red and blue solid lines. This is because even when no contracts are set in local currency, it is still worthwhile to denominate contracts in this currency if the policy risk is low enough. As more contracts are signed in local currency, its attractiveness increases as the government endogenously uses inflation to redistribute resources more effectively.

When policy risk is intermediate (case 2) we have multiple equilibria. Multiplicity arises due to the complementarities between private and government actions. As more contracts are set in local currency, the government uses inflation to provide more insurance through better redistribution. One of the equilibria involves full use of foreign currency. If all private contracts are set in foreign currency, there are no incentives for the government to use inflation in order to redistribute resources. Therefore, the marginal benefit of local currency is given only by policy risk, which in this region is higher than the price risk of foreign currency. Another equilibrium involves full use of local currency. If all private contracts are denominated in local currency, then the government is incentivized to use inflation to redistribute resources efficiently, and this makes local currency more attractive than foreign currency. Finally, there is a third interior equilibrium in which the level of B_l is such that the marginal benefits of local and foreign currencies are equal. In the figure, the three equilibria correspond to the three intersections of the blue and the middle red dashed line.

Finally, when policy risk is high enough (case 3) the unique equilibrium involves full use of foreign currency. This equilibrium exists because the marginal benefit of local currency is completely determined by policy risk when all contracts are set in foreign currency, and policy risk is larger than the price risk of foreign currency. The equilibrium is unique because even if all contracts are set in local currency, the government's use of inflation to redistribute resources does not compensate for the high level of policy risk. In the figure, this case corresponds to the intersection of the lowermost red dashed line with the blue line.

This characterization helps rationalize observed differences in the use of foreign currency as a unit of account across countries. In particular, it offers a rationalization for why countries with low levels of policy risk, such as the U.S., Germany, and Japan, rely exclusively on their local currency as a unit of account in domestic contracts. In contrast, countries with high policy risk, such as those in Latin America and Eastern Europe, tend to partially or fully rely on foreign currency as a unit of account.

2.6 Equilibrium Selection

We now consider a global games refinement to uniquely select an equilibrium of the model above. This is useful as it allows for sharper predictions of model behavior as well as a cleaner comparison with the constrained efficient allocation in the next subsection.

We consider a variant of the model described above in which agents receive a noisy signal of fundamentals, and analyze the limiting case in which the noise is zero. In particular, we assume that in period 1, all buyer-seller pairs receive a noisy signal of the local policy risk, $\xi \equiv \mathbb{E}\left[\hat{\phi}\right]$.[11] Private agents have a common uniform prior over ξ with support $\left[\underline{\xi}, \overline{\xi}\right]$. Let i index each buyer-seller pair with $i \in [0,1]$. Then, pair i receives a signal

$$\hat{\xi}_i = \xi + \varepsilon_i,$$

where $\varepsilon_i \sim U\left[-\eta, \eta\right]$ is uniformly distributed and is independent across all i. We assume that the support of $\hat{\phi}$ is common knowledge across all agents with

$$\underline{\hat{\phi}} \leqslant \underline{\xi} < \overline{\xi} \leqslant \overline{\hat{\phi}}.$$

We now show that the optimal private contract satisfies a simple cutoff property.

Lemma 1. *Suppose that Assumptions 1 and 2 hold, and fix some $\xi \in \left(\underline{\xi}, \overline{\xi}\right)$. Then, for η small enough there exists a threshold ξ^* such that the optimal choice of payments in local currency is given by*

$$b_l\left(\hat{\xi}\right) = \begin{cases} 0 & \hat{\xi} < \xi^* \\ \frac{y}{\overline{\phi}_l^{**}} & \hat{\xi} > \xi^* \end{cases}$$

where $\overline{\phi}_l^{**}$ is the positive solution to

$$\overline{\phi}_l^{**} = \overline{\hat{\phi}} + \frac{1}{2}\frac{1}{\psi}\left(\overline{\theta}_s - \underline{\theta}_b\right)\frac{y}{\overline{\phi}_l^{**}}.$$

The lemma extends the results of Proposition 1 when there are information asymmetries. The private currency choice in this case depends on the signal of policy risk. When this signal realization is large (low policy risk), agents perceive a high marginal benefit of the local currency and denominate contracts in it.

Given the characterization of the individual contract, the following proposition characterizes the unique competitive equilibrium in the limiting case in which $\eta \to 0$.

Proposition 3. *Suppose that Assumptions 1 and 2 hold. Then, for $\eta \to 0$, there exists a threshold μ_{GG} such that:*

1. *If $\mathbb{E}\left[\hat{\phi}\right]/\overline{\hat{\phi}} > \mu_{GG}$, there exists a unique equilibrium in which $B_l = y/\overline{\phi}_l^{**}$.*
2. *If $\mathbb{E}\left[\hat{\phi}\right]/\overline{\hat{\phi}} \leqslant \mu_{GG}$, there exists a unique equilibrium in which $B_l = 0$.*

Additionally, $\mu_2 < \mu_{GG} < \mu_1$.

As in the global games literature (see, for example, Morris and Shin, 2001), the introduction of dispersed signals gives rise to uncertainty about the agents' actions and, therefore, attenuates the source of strategic interaction. In this case, the uncertainty causes

[11] We assume that $\overline{\hat{\phi}}$ is common knowledge and, hence, a signal of $\mathbb{E}\left[\hat{\phi}\right]$ constitutes a signal of policy risk $\mathbb{E}\left[\hat{\phi}\right]/\overline{\hat{\phi}}$.

agents to perceive a lower aggregate B_l and, thus, anticipate lower insurance benefits from the government's monetary policy. This attenuates the complementarities and yields a unique equilibrium. This equilibrium satisfies a cutoff property: if policy risk is large (i.e., $\mathbb{E}\left[\hat{\phi}\right]/\overline{\hat{\phi}} < \mu_{GG}$), there is a unique equilibrium in which all contracts are denominated in foreign currency, while if policy risk is small (i.e., $\mathbb{E}\left[\hat{\phi}\right]/\overline{\hat{\phi}} > \mu_{GG}$), there is a unique equilibrium in which all contracts are denominated in local currency. Thus, economies with higher policy risk are more likely to use foreign currency to denominate contracts. The cutoff is in the region in which there are multiple equilibria in the economy with full information. Therefore, the global games perturbation selects one of the extreme equilibria (full use of either foreign or local currency) as the unique equilibrium in this range of policy risk. Figure 2 illustrates the set of equilibria, with and without the global games refinement, as a function of policy risk.

2.7 Constrained Efficiency

We now consider the problem of a social planner who chooses allocations subject to the same constraints that private agents face and the same choice of monetary policy made by the government in the second period. The utilitarian social planner solves

$$\max_{C_s, C_b, B_l, B_f, \phi_l} \mathbb{E}\left(-x + \theta_s C_s + (1+\lambda)x + \theta_b C_b - l(\phi_l)\right)$$

subject to the definitions of C_b and C_s in (6) and (7), respectively, the participation constraints of the buyer (3) and seller (2), the payments feasibility constraint (1), and the best responses of the government (8), and (9).

Analogously to the competitive equilibrium, the following proposition characterizes the solution to the planner's problem for different values of policy risk, and shows that the efficient allocation involves the full use of foreign currency when policy risk is high and the full use of local currency when policy risk is low.

Proposition 4. *Suppose that Assumptions 1 and 2 hold. Then, there exists a threshold μ_{SP}, with $\mu_2 < \mu_{SP} < \mu_1$, such that:*

1. *If $\mathbb{E}\left[\hat{\phi}\right]/\overline{\hat{\phi}} \geqslant \mu_{SP}$, then the solution to the social planner's problem is $B_l^{sp} = y/\overline{\phi}_l^*$, where $\overline{\phi}_l^*$ was defined in Proposition 2.*

2. *If $\mathbb{E}\left[\hat{\phi}\right]/\overline{\hat{\phi}} \leqslant \mu_{SP}$, then the solution to the social planner's problem is $B_l^{sp} = 0$.*

The proof follows from the observation that Assumption 2 implies that the social planner's problem is strictly convex. As a result, computing the solution of this problem involves comparing the values of the objective at end-points. The relative value of these end-points depends on whether policy risk is high or low. Intuitively, a low policy risk increases the value of the full local currency equilibrium relative to the full foreign currency one, while a high policy risk does the opposite.

This result also shows that an interior equilibrium can never be efficient. In particular, for policy risk within the range (μ_{sp}, μ_1), the full local currency equilibrium dominates the interior and full foreign currency equilibria, while for policy risk within (μ_2, μ_{sp}) the full foreign currency equilibrium dominates the other two equilibria. In contrast, if policy risk is either very low or very high, the unique competitive equilibrium (full local in the former, full foreign in the latter) is constrained efficient.

To understand why $\mu_2 < \mu_{SP} < \mu_1$, suppose first that policy risk equals $\mu_1 = \mathbb{E}[\phi_f] / \overline{\phi}_f$. At this point, the price risk of the local and foreign currency is identical if all contracts are denominated in foreign currency. However, denominating contracts in local currency carries an additional private net insurance benefit (a higher covariance net of price risk) and an additional cost associated with deviating from the inflation target. Assumption 2 guarantees that the net insurance benefits are higher than the inflation costs. Consequently, we show that from the planner's perspective, the full local currency allocation dominates the full foreign currency one. Therefore, it must be that $\mu_{SP} < \mu_1$. To see why an equilibrium with foreign currency use can exist in the region (μ_{sp}, μ_1), note that since private agents are infinitesimal, their actions do not affect the policy choice of the government. As a result, if all agents denominate contracts in foreign currency, a particular buyer-seller pair has no incentive to denominate contracts in local currency since it is associated with higher price risk and no insurance (since $B_l = 0$).

Next, suppose that policy risk equals μ_2. At this point, if all agents are denominating contracts in local currency, the private marginal benefits of denominating contracts in either currency is identical. However, there is an additional cost associated with having the price level deviate from its target which is internalized only by the planner. As a result, the planner strictly prefers to denominate all contracts in foreign currency, which implies that $\mu_{SP} > \mu_2$. However, for policy risk in the range (μ_2, μ_{sp}) an equilibrium with local currency use can exist because the private marginal benefit of denominating contracts in local currency is larger than that of denominating contracts in foreign currency if all agents denominate contracts in local currency. In particular, private agents do not internalize these inflation costs.

The combination of the equilibrium characterization and the above result helps rationalize some of the policies described in the introduction. Consider a country with very low policy risk. The model predicts that contracts signed within the country will be denominated in local currency and it is efficient to do so. For slightly higher levels of policy risk, equilibria in which contracts are denominated in foreign currency exist but are inefficient. Optimal regulation should prescribe limits on the use of foreign currency to denominate contracts. This might help explain the prevalence of policies that regulate the use of foreign currency in a variety of emerging economies. For example, Brazil and Colombia implemented policies that forced de-dollarization of contracts by restricting the denomination of bank deposits or loans in foreign currency (Galindo and Leiderman, 2005). Similarly, Turkey and Hungary imposed restrictions on the access to foreign currency loans and mortgages, respectively (de Crescenzio et al., 2015). In contrast, for high enough levels of policy risk, optimal regulation should encourage and incentivize the use of foreign currency. An example of this type of policies is the forced dollarization adopted by Ecuador in the year 2000.

The existence of a region in which the competitive equilibrium and social planner's solution do not coincide, thus warranting regulation, depends crucially on modeling both price risk (arising from policy risk) and insurance benefits jointly. To illustrate this point, assume that $\theta_s = \theta_b = 1$ so there are no insurance benefits of the local currency. Then, private agents choose the currency with lower price risk and this is also the efficient outcome. Alternatively, assume that there is no policy risk and no foreign currency risk. Then, given Assumption 2, all agents choose local currency contracts (assuming that they do so if indifferent) and this is also efficient. Consequently, there is no room for regulating private contracts if only one of these forces is studied in isolation.

The global games approach described in the previous section allows for a cleaner comparison between the equilibrium and the efficient allocation.[12] Given the characterization of the competitive equilibrium and social planner's problem, we see that because of equilibrium multiplicity, identical fundamentals can be consistent with outcomes that are efficient or inefficient. Therefore, the precise implication for currency choice regulation is unclear. In contrast, with the global games approach, we can have definitive predictions based solely on fundamentals as to when and what type of regulation is needed. We find that, depending on fundamentals, the global games equilibrium can feature either underuse or overuse of local currency. We also show that if the fundamentals satisfy a slightly stronger version of Assumption 2, then the only inefficiency that exists is the one in which there is underuse of local currency. The following proposition provides a complete characterization of this result.

Proposition 5. *Suppose that Assumptions 1 and 2 hold. Then,*

$$\mu_{GG} - \mu_{SP} \propto \Delta,$$

[12]Note that, as is standard practice, we are using the global games refinement as an equilibrium selection device. Thus, the underlying environment is still one in which there is full information about fundamentals. Consequently, we can compare this global games equilibrium with the original planning problem in which there are no informational asymmetries.

where

$$\Delta \equiv \iota\left(\tilde{\kappa}_2 - \tilde{\kappa}_1\right) - \frac{1}{2}\mathbb{E}\left(\left(\theta_s - \theta_b\right)^2\right)$$

and ι, $\tilde{\kappa}_2$ *and* $\tilde{\kappa}_1$ *are constants defined in the Appendix. Then,*

- *If* $\Delta \geqslant 0$, $\mu_{GG} \geqslant \mu_{SP}$ *and thus for* $\mathbb{E}\left[\hat{\phi}\right]/\overline{\hat{\phi}} \in (\mu_{SP}, \mu_{GG})$ *all equilibrium contracts are denominated in foreign currency, while the constrained-efficient allocation calls for all contracts to be denominated in local currency. Outside of this interval, the choice of currency in the equilibrium and that of the planner coincide.*

- *If* $\Delta < 0$, $\mu_{GG} < \mu_{SP}$ *and thus for* $\mathbb{E}\left[\hat{\phi}\right]/\overline{\hat{\phi}} \in (\mu_{GG}, \mu_{SP})$ *all equilibrium contracts are denominated in local currency, while the constrained-efficient allocation calls for all contracts to be denominated in foreign currency. Outside of this interval, the choice of currency in the equilibrium and that of the planner coincide.*

Recall that μ_{GG} is the policy risk threshold selected by the global games approach above which all equilibrium contracts are denominated in local currency and below which all equilibrium contracts are denominated in foreign currency. The proposition provides a comparison between μ_{GG} and μ_{SP}. It shows that, depending on parameter values, μ_{GG} can be either higher or lower than μ_{SP}.

To understand this result, consider the following decomposition

$$\mu_{GG} - \mu_{SP} = \underbrace{\iota\left(\tilde{\kappa}_2 - \tilde{\kappa}_1\right)}_{\propto(\mu_{GG}-\mu_2)} - \underbrace{\frac{1}{2}\mathbb{E}\left(\left(\theta_s - \theta_b\right)^2\right)}_{\propto(\mu_2-\mu_{SP})}.$$

The first term on the right-hand side is proportional to $\mu_{GG} - \mu_2$, which captures the difference between the global games equilibrium and the competitive equilibrium. The main difference is that, for a given level of policy risk, the perceived marginal benefit of denominating contracts in local currency is lower in the global games equilibrium. This is because we are considering the noiseless limit of a model in which agents receive heterogeneous signals and thus perceive lower aggregate payments in local currency.[13] Thus, the level of policy risk required for an agent in the global games equilibrium to be indifferent between denominating a contract in local and foreign currency is lower than in the

[13] To understand why, recall that the aggregate payments in the baseline model under the full local currency equilibrium is $B_l = y/\overline{\phi}_l^*$. In the global games environment, a buyer-seller pair believes that the aggregate payments in local currency are $B_l^{GG} = \left(1 - \Pr\left\{\xi \leqslant \xi^* \mid \hat{\xi}\right\}\right) y/\overline{\phi}_l^{**}$, where ξ^* was defined in Lemma 1, $\hat{\xi}$ is the true fundamental, and $\overline{\phi}_l^{**}$ is the maximal price level in the global games equilibrium. If $\left(1 - \Pr\left\{\xi \leqslant \xi^* \mid \hat{\xi}\right\}\right)/\overline{\phi}_l^{**} < 1/\overline{\phi}_l^*$ (which is true in equilibrium), then $B_l^{GG} < y/\overline{\phi}_l^*$. Since the marginal benefit of denominating contracts in local currency is increasing in B_l, we have that $M_l\left(B_l^{GG}\right) < M_l\left(y/\overline{\phi}_l^*\right)$. Consequently, private agents in the global games environment perceive a lower marginal benefit of denominating contracts in local currency.

competitive equilibrium. This pushes μ_{GG} to be greater than μ_2. Next, the second term is proportional to $\mu_2 - \mu_{SP}$, which captures the difference between the competitive equilibrium and the social planner's allocation. We have already seen that $\mu_2 - \mu_{SP} > 0$ because private agents do not internalize the inflation costs. Consequently, whether $\mu_{GG} > \mu_{SP}$ or not depends on whether this reduction in the net private marginal benefits is larger than the inflation losses internalized only by the planner. The condition guaranteeing that $\mu_{GG} \geqslant \mu_{SP}$ (i.e., $\Delta > 0$) is a slight strengthening of Assumption 2. To see this, consider the case in which θ_s and θ_b are i.i.d., in which case Assumption 2 is given by (10). The condition guaranteeing that $\Delta > 0$ is

$$\operatorname{var}(\theta) \geqslant \frac{\lambda}{\left[2\left(1-\frac{1}{2\iota}\right)+\lambda\right]}\mathbb{E}\left[\theta\right]\left(\frac{\mathbb{E}\left[\phi_f\right]}{\overline{\phi}_f}\left(\overline{\theta}-\underline{\theta}\right)\right) \tag{11}$$

where $\iota < 1$. Therefore, if the insurance benefits are large enough, the region of policy risk in which the equilibrium use foreign currency is low relative to the efficient allocation is not robust to an informational perturbation.

Given this result, consider a government that can regulate the currency choice of private contracts. In particular, it can force private agents to use either local or foreign currency. If (11) holds, then for an intermediate range of policy risk it is optimal to force private agents to denominate contracts in local currency. If, (11) does not hold, then there exists a range of policy risk for which it is optimal to force private agents to denominate contracts in foreign currency.[14]

Figure 2 summarizes the set of equilibria and constrained efficient allocations for all possible values of policy risk for the case in which $\Delta > 0$.

3 Applications and Extensions

In this section, we extend the model to study three applications of the theory. In Section 3.1, we study a model in which the role of monetary policy is to reduce default costs. In Section 3.2, we introduce international trade into our model and study how the equilibrium use of foreign currency changes. Finally, in Section 3.3 we show how our model can generate the observed hysteresis in the use of foreign currency.

[14]Note that for values $\mathbb{E}\left[\hat{\phi}\right]/\overline{\hat{\phi}} \geqslant \max\{\mu_{GG}, \mu_{SP}\}$, even though the choice of currency in the global games and that in the planner's solution coincide, the sizes of these contracts do not. In particular, private agents choose a lower B_l than the planner due to the informational friction. Consequently, in this interval, the planner would like to subsidize the use of local currency.

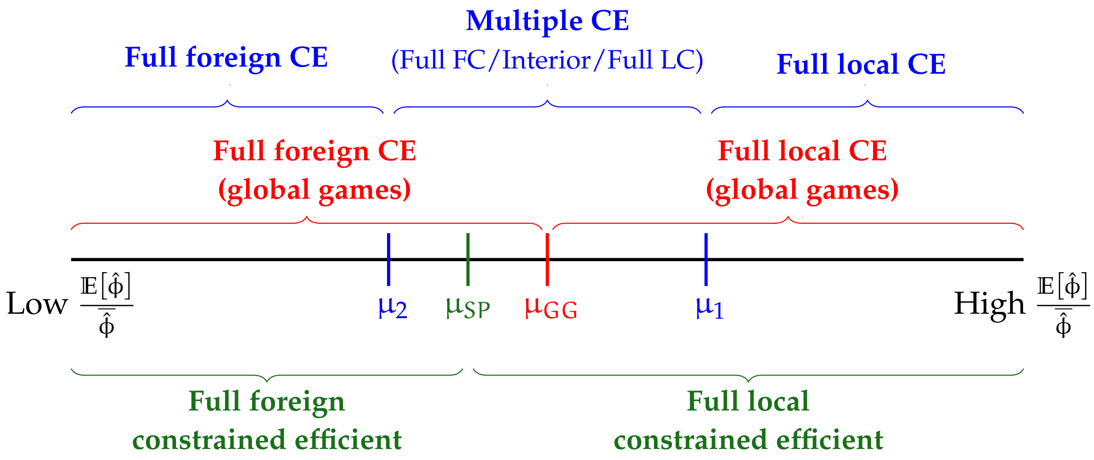

Figure 2: Equilibrium set and efficient allocations for different levels of policy risk when $\Delta > 0$

3.1 A Model with Strategic Default

In this section, we introduce a model with strategic default in which the role of policy is to minimize the costs associated with default, and show that this model maps directly into the baseline setup. We do this to argue that the environment with taste shocks described before is quite flexible and encompasses other interesting environments. Moreover, this analysis shows that the main results continue to hold even if we allow private agents to introduce some degree of state contingency in contracts.

Consider a model similar to the baseline in which buyers and sellers are no longer subject to explicit taste shocks. However, there is still uncertainty about future prices. We allow buyers to fully default on their obligations in period 2 and suffer a cost proportional to the level of defaulted debt. In particular, a buyer defaulting on payments (b_l, b_f) obtains a utility of

$$y - \chi\left(\phi_l b_l + \phi_f b_f\right),$$

where $\chi\left(\phi_f b_f + \phi_l b_l\right)$ is the utility cost of default, which depends on the *level* of defaulted debt. This implies that a buyer who defaults on a larger stock of debt suffers a higher cost. One interpretation of this cost is that if there is exclusion after default, the exclusion time depends positively on the level of defaulted debt (see Kirpalani (2016), who shows the optimality of such punishments in a model with endowment risk, and Cruces and Trebesch (2013), who document in the sovereign default data that higher haircuts are associated with longer periods of exclusion). Assume that χ is a random variable with cdf $F_\chi(\cdot)$ and bounded support $\chi \in [\underline{\chi}, \overline{\chi}]$, with $\underline{\chi} < 1 < \overline{\chi}$. Therefore, since the value of not defaulting is $y - (\phi_l b_l + \phi_f b_f)$, the buyer defaults if $\chi < 1$.

The contracting problem is

$$\max_{x,b_l\geqslant 0,b_f\geqslant 0}(1+\lambda)\,x+\mathbb{E}\left[(y-\phi_l b_l-\phi_f b_f)\,\mathbb{I}_{\chi\geqslant 1}+(y-\chi\,(\phi_l b_l+\phi_f b_f))\,\mathbb{I}_{\chi<1}\right]$$
$$-x+\mathbb{E}\left[y+(\phi_l b_l+\phi_f b_f)\,\mathbb{I}_{\chi\geqslant 1}\right]$$

subject to the participation constraint of the seller,

$$-x+\mathbb{E}\left[y+(\phi_l b_l+\phi_f b_f)\,\mathbb{I}_{\chi\geqslant 1}\right]\geqslant y,$$

the participation constraint of the buyer,

$$(1+\lambda)\,x+\mathbb{E}\left[(y-\phi_l b_l-\phi_f b_f)\,\mathbb{I}_{\chi\geqslant 1}+(y-\chi\,(\phi_l b_l+\phi_f b_f))\,\mathbb{I}_{\chi<1}\right]\geqslant y,$$

and the non-negativity constraint on buyer's consumption.[15] Next, we consider the government's problem. Clearly, if $\chi>1$, then the government's optimal choice involves setting $\phi_l=\hat{\phi}$ since buyers repay their debt and there is no motive to deviate from the inflation target. If instead $\chi<1$, then the government's problem is

$$\max_{\phi_l}-\chi\,(\phi_l B_l+\phi_f B_f)-l\,(\phi_l)\,,$$

which, given the functional form of the loss function, implies that the optimal choice of ϕ_l satisfies

$$\phi_l=\hat{\phi}-\frac{1}{\psi}\chi B_l.$$

In this case, the government optimally chooses to increase inflation more than its target to lower the burden of default for buyers. The higher the use of local currency in contracts, B_l, the higher is the optimal inflation (lower ϕ_l) chosen by the government. Also, note that here the maximal price level is given by

$$\overline{\phi}_l=\overline{\hat{\phi}}.$$

Next, we show how this setup can be mapped into the model described in the previous section. Define

$$\theta_s=\begin{cases}0 & \text{if } \chi<1\\ 1 & \text{if } \chi\geqslant 1\end{cases}\tag{12}$$

[15]Note that given our default specification, the non-negativity constraint on the buyer's consumption is equivalent to imposing the payments feasibility constraint in the states in which the buyer chooses to repay.

and

$$\theta_b = \begin{cases} \chi & \text{if } \chi < 1 \\ 1 & \text{if } \chi \geqslant 1. \end{cases} \tag{13}$$

Given this mapping, we have that

$$(1+\lambda)\,\theta_s - \theta_b = \begin{cases} -\chi & \text{if } \chi < 1 \\ \lambda & \text{if } \chi \geqslant 1. \end{cases}$$

The next proposition shows that the set of equilibrium outcomes of the taste-shock model with the above processes is identical to the equilibrium outcomes of the default model. Moreover, we show that under a sufficient condition, the implied taste-shock environment satisfies Assumptions 1 and 2. Therefore, we can apply all the previous results to the model with default.

Proposition 6. *The set of equilibrium outcomes of the taste-shock model implied by (12) and (13) is identical to that of the model with default. Suppose further that*

$$\lambda\,(1 - F_\chi\,(1)) - F_\chi\,(1)\,\mathbb{E}\,[\chi \mid \chi \leqslant 1] > 0.$$

Then, the taste-shock model implied by (12) and (13) satisfies Assumptions 1 and 2.

To understand the above result, notice that since the seller gets nothing in the default state, its payoffs are identical to those in the taste-shock model in which $\theta_s = 0$. The buyer's payoffs in the default state are identical (up to a constant) to those in the taste-shock model in which $\theta_b = \chi < 1$. We can similarly construct values for the taste shocks so that the payoffs of both buyer and seller coincide in the no-default states as well. Consequently, the proposition implies that the equilibrium characterization and the efficiency results are identical to those found in the baseline model. In particular, the optimal currency choice trades off price risk and the covariance benefits. The latter arises here from the reduction in default costs when inflation is high. Moreover, the model features complementarities in private and government's actions: the larger B_l is, the greater the incentive to use policy to reduce default costs. Finally, note that the assumption in the proposition is the analogue to Assumption 1 and states that the expected gains from trade are larger than the expected costs of default.

Another important takeaway from this model is that that observed inflation policy need not always reflect a redistributive motive. Indeed, during normal times, when there is no risk of default (when $\chi > 1$), inflation is set at its target ($\phi_l = \hat{\phi}$). However, in times of crises, when there is default ($\chi < 1$), the government chooses inflation to reduce the burden of default.

3.2 Contracts in International Trade

One of the facts mentioned in the introduction is that there is extensive use of the U.S. dollar as a unit of account in international trade contracts. Trade involving countries with seemingly low policy risk is often invoiced in dollars. For example, Japan has low levels of policy risk and domestic dollarization, and yet has a significant fraction of trade contracts denominated in dollars. In this section, we study an extension of our baseline model with international trade that helps rationalize these facts. We incorporate international trade in our model by studying an economy in which agents from one country also trade with agents from another country, and contracts can be set in any of the currencies of the involved countries or in a third, external currency. Our main result in this section shows that contracts between agents located in different countries are more likely to use foreign currency as compared with contracts signed by agents in the same country.

The extended setup of the model is as follows. There are two countries, denoted by i and j, which are symmetric. In each country there is a continuum of buyers and sellers of equal size. Within each country, the taste shocks of buyers and sellers are distributed in an identical fashion to the baseline model. In addition, we assume that these shocks are independent across countries. A contract between a buyer and a seller consists of the provision of a special good in exchange for the promise of future payment. The first difference with the baseline model is that a fraction γ of contracts signed by buyers in a country has an international counterparty, while a fraction $1-\gamma$ has a domestic counterparty (in what follows, we refer to the fraction of international contracts γ as the degree of openness in a country). The second difference is that we allow contracts to be set in three possible units of account: currencies from country i and j, and the foreign currency f. The price levels of currencies i and j (denoted by ϕ_i and ϕ_j, respectively) are chosen by the governments of each country, whereas the price of the foreign currency is exogenous.

The optimal private contract between a buyer and a seller from the same country is identical to that characterized in the baseline model. Next, we characterize the optimal international contract. Let $\tilde{x}_i$ be the amount of special good provided by a seller from country j to a buyer of country i and $\tilde{b}_{ic}$ be the promised payment of a buyer from country i in currency c to a seller in country j.[16] The optimal private contract between a buyer in country i and a seller in country j solves

$$\max_{\tilde{x}_i, \tilde{b}_{ii} \geqslant 0, \tilde{b}_{ij} \geqslant 0, \tilde{b}_{if} \geqslant 0} (1+\lambda)\, \tilde{x}_i + \mathbb{E}\left[\theta_{ib}\left(y - \phi_i \tilde{b}_{ii} - \phi_j \tilde{b}_{ij} - \phi_f \tilde{b}_{if}\right)\right]$$
$$-\tilde{x}_i + \mathbb{E}\left[\theta_{js}\left(y + \phi_i \tilde{b}_{ii} + \phi_j \tilde{b}_{ij} + \phi_f \tilde{b}_{if}\right)\right]$$

[16]Note that we have suppressed the dependency on j, since knowing that the buyer is located in country i implies that the seller is from country j.

subject to the participation constraint of the seller,

$$-\tilde{x}_i + \mathbb{E}\left[\theta_{js}\left(y + \phi_i \tilde{b}_{ii} + \phi_j \tilde{b}_{ij} + \phi_f \tilde{b}_{if}\right)\right] \geqslant \mathbb{E}\left[\theta_{js} y\right],$$

the participation constraint of the buyer,

$$(1+\lambda)\,\tilde{x}_i + \mathbb{E}\left[\theta_{ib}\left(y - \phi_i \tilde{b}_{ii} - \phi_j \tilde{b}_{ij} - \phi_f \tilde{b}_{if}\right)\right] \geqslant \mathbb{E}\left[\theta_{ib} y\right],$$

and the payments feasibility constraint,

$$\phi_i \tilde{b}_{ii} + \phi_j \tilde{b}_{ij} + \phi_f \tilde{b}_{if} \leqslant y, \tag{14}$$

for all possible price realizations, where θ_{ib} and θ_{js} denote the taste shocks of the buyer from country i and the seller from country j, respectively. The solution to this problem is characterized in Lemma 2 in the Appendix, and is similar to Proposition 1. Taking prices as given, agents write contracts using the currency that has the largest marginal benefit, allowing for combinations of two or three currencies whenever the buyer is indifferent.

Next, we revisit the government's problem. There are two utilitarian governments that control monetary policy and choose the price level of the local currencies in countries i and j. We assume that both countries have the same level of policy risk, $\mathbb{E}\left[\hat{\phi}_i\right]/\overline{\hat{\phi}}_i = \mathbb{E}\left[\hat{\phi}_j\right]/\overline{\hat{\phi}}_j$. This allows us to compare the equilibrium outcomes of the two-country model with those of the baseline model. Denote by $\tilde{B}_{ic}$ the aggregate promised payments in currency c from buyers in country i to sellers in country j. Similarly, denote by B_{ic} the aggregate promised payments in currency c from domestic contracts in country i. The problem of the government in country i is given by

$$\max_{\phi_i} \theta_{ib} C_{ib} + \theta_{is} C_{is} - l\left(\phi_i\right),$$

where the aggregate consumption of buyers is given by

$$C_{ib} = y - \gamma\left(\phi_i \tilde{B}_{ii} + \phi_j \tilde{B}_{ij} + \phi_f \tilde{B}_{if}\right) - (1-\gamma)\left(\phi_i B_{ii} + \phi_j B_{ij} + \phi_f B_{if}\right), \tag{15}$$

and that of sellers is given by

$$C_{is} = y + \gamma\left(\phi_i \tilde{B}_{ji} + \phi_j \tilde{B}_{jj} + \phi_f \tilde{B}_{jf}\right) + (1-\gamma)\left(\phi_i B_{ii} + \phi_j B_{ij} + \phi_f B_{if}\right).$$

Given our functional form assumption for the inflation loss function, the solution to the problem of the government in country i is

$$\phi_i = \hat{\phi}_i + \frac{1}{\psi}\left[\gamma\left(\theta_{is}\tilde{B}_{ji} - \theta_{ib}\tilde{B}_{ii}\right) + (1-\gamma)\left(\theta_{is} - \theta_{ib}\right)B_{ii}\right], \quad (16)$$

and the largest feasible price level is

$$\overline{\phi}_i = \overline{\hat{\phi}} + \frac{1}{\psi}\left[\gamma\left(\overline{\theta}_{is}\tilde{B}_{ji} - \underline{\theta}_{ib}\tilde{B}_{ii}\right) + (1-\gamma)\left(\overline{\theta}_{is} - \underline{\theta}_{ib}\right)B_{ii}\right].$$

The problem of the government in country j is symmetric. We now define a competitive equilibrium with international trade. For ease of notation, define $\pi_i \equiv \left(\tilde{x}_i, \tilde{b}_{ii}, \tilde{b}_{ij}, \tilde{b}_{if}, x_{ii}, b_{ii}, b_{ij}, b_{if}\right)$ to be the vector of private choices in country i and $\Pi_i \equiv \left(\tilde{B}_{ii}, \tilde{B}_{ij}, \tilde{B}_{if}, B_{ii}, B_{ij}, B_{if}\right)$, to be the vector of aggregate choices.

Definition 2. *Given a degree of openness γ, a competitive equilibrium is an allocation for private citizens in each country, (π_i, π_j), aggregate promised payments (Π_i, Π_j), and inflation choices for governments ϕ_i and ϕ_j such that: 1. Given ϕ_i and ϕ_j , the private allocations solve the contracting problems defined in (4) and (14), 2. Given Π_i, ϕ_i satisfies (16) (and similarly for country j), and 3. Aggregate choices are consistent with private ones, $b_{ik} = B_{ik}$, $\tilde{b}_{ik} = \tilde{B}_{ik}$, $b_{jk} = B_{jk}$, and $\tilde{b}_{jk} = \tilde{B}_{jk}$ for $k \in \{i, j, f\}$.*

We restrict attention to symmetric equilibria in which all international trade contracts are set in the same currency, i.e., $\tilde{B}_{jc} = \tilde{B}_{ic} \equiv \tilde{B}_c$ for all c. In Appendix C.6, we relax this assumption and also consider asymmetric equilibria.

In the following propositions, we argue that foreign currency is more likely to be used in international contracts than in domestic contracts. For expositional purposes, we first focus on the case with international contracts only (that is, $\gamma = 1$) and compare the outcomes to that in the baseline economy with only domestic contracts. Recall that μ_2 is the threshold from the baseline economy such that if $\mathbb{E}\left[\hat{\phi}\right]/\overline{\hat{\phi}} < \mu_2$ (i.e., policy risk is large enough), then there is a unique equilibrium in which only foreign currency is used as a unit of account.

Proposition 7. *Suppose that Assumptions 1 and 2 hold and $\gamma = 1$. Then, there exists a threshold μ_2^I such that if $\mathbb{E}\left[\hat{\phi}_i\right]/\overline{\hat{\phi}}_i = \mathbb{E}\left[\hat{\phi}_j\right]/\overline{\hat{\phi}}_j \leqslant \mu_2^I$, there exists a unique symmetric equilibrium in which $\tilde{B}_i = \tilde{B}_j = 0$. Furthermore, $\mu_2^I > \mu_2$.*

The threshold μ_2^I depends on parameters and is defined in (26) in the Appendix. As in the baseline model, there exists a threshold μ_2^I such that if policy risk in country i and j is larger than that implied by this threshold, the unique equilibrium displays the use of foreign currency as the sole unit of account. However, the most important result of this proposition is that $\mu_2^I > \mu_2$; that is, the threshold obtained in the two-country model is larger than the one found in the baseline model. This implies that for levels of policy risk such that $\mu_2^I > \mathbb{E}\left[\hat{\phi}_i\right]/\overline{\hat{\phi}}_i = \mathbb{E}\left[\hat{\phi}_j\right]/\overline{\hat{\phi}}_j > \mu_2$, there exists a unique foreign currency

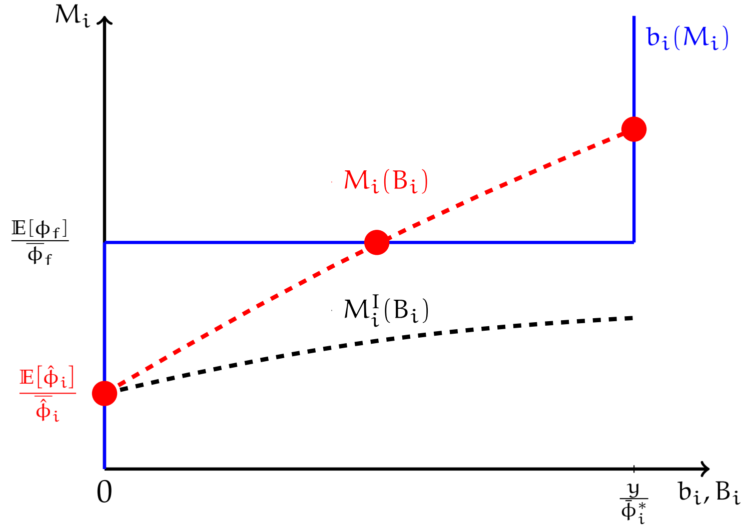

Figure 3: Comparing currency choice in international versus domestic contracts

equilibrium in the model with international trade, while there can exist equilibria with local currency in the model with only domestic contracts. This result suggests that we are more likely to observe international trade contracts denominated in foreign currency than domestic contracts denominated in such currency.

The intuition behind this result is that in the case of international contracts, each government finds it optimal to use inflation to respond only to the taste shocks of its own citizens and not to those of the other country's citizens. That is, governments do not react to the taste shocks of foreign buyers or sellers, which implies that the covariance term in equation (5) is lower for a given aggregate exposure to the local currency. This, in turn, lowers the marginal benefit of using local currencies of either country, and makes foreign currency more attractive for private contracts. This is shown in Figure 3, which illustrates the set of equilibria in the economies with domestic and international contracts, for a level of policy risk in between μ_2 and μ_2^I, when $\mathbb{E}[\theta_s] = \mathbb{E}[\theta_b] = 1$ and $\lambda = 1$. As in the baseline model, the blue line corresponds to the individual optimal promised payment denominated in currency i for a given government policy and, thus, for a given M_i. The red line denoted by $M_i(B_i)$ depicts the marginal benefit of currency i as a function of B_i in the economy with domestic contracts, whereas the dotted black line denoted by $M_i^I(B_i)$ corresponds to the marginal benefit of currency i as a function of B_i in the economy with international contracts. In the latter, the lower covariance term implies a lower slope of the black dotted line, which eliminates the equilibrium with full use of local currency.

While the proposition focuses on symmetric equilibria, in Appendix C.6 we argue that the uniqueness result generalizes to all equilibria under a slightly stronger parametric assumption.

Next, we study the use of foreign currency as a unit of account when both domestic and international contracts are present (i.e., when $\gamma < 1$).

Proposition 8. *Suppose that Assumptions 1 and 2 hold. For γ low enough, there exists a threshold $\mu_2(\gamma)$ such that if $\mathbb{E}[\hat{\phi}_i]/\bar{\hat{\phi}}_i = \mathbb{E}[\hat{\phi}_j]/\bar{\hat{\phi}}_j \in (\mu_2(\gamma), \mu_2^I)$, then:*

- *In any equilibrium, international contracts are denominated in foreign currency (i.e., $\tilde{B}_i = \tilde{B}_j = 0$).*
- *There exists an equilibrium in which domestic contracts are denominated in local currency (i.e., $B_i = B_j > 0$).*
- *The threshold $\mu_2(\gamma)$ is increasing in γ.*

The first two points of the proposition state that for a given degree of openness below a threshold, there exists an interval of policy risk—$(\mu_2(\gamma), \mu_2^I)$—such that all equilibria feature international contracts denominated only in the foreign currency. However, there exist equilibria in which domestic contracts are denominated in the local currency. In this interval, the government provides enough insurance to justify the choice of local currency in domestic contracts but not in foreign contracts, because the government responds only to the taste shocks of its own citizens. The last point of the proposition states that the interval of policy risk in which such equilibria exist shrinks as the degree of openness increases. As the economy becomes more open (i.e., γ becomes larger), the share of international contracts denominated in the foreign currency increases, which reduces the government's incentives to provide insurance and induces domestic contracts to also be denominated in foreign currency. Thus, more open economies are more likely to exhibit both domestic and international contracts denominated in foreign currency. This model prediction is supported by empirical evidence that shows that a country's degree of trade openness (as measured by the ratio of exports and imports to GDP) is positively associated with financial dollarization (see, e.g., Nicolo et al. (2003a), Calvo-Gonzalez et al. (2007), and Rosenberg and Tirpák (2008)).

3.3 Hysteresis

A distinctive feature among many emerging economies is the hysteresis of dollarization even after inflation risk stabilized. For example, the hyperinflation in Argentina during the late 1980s prompted an increase in the share of deposits in dollars from 40% to near 100%. However, despite the large success of governments in stabilizing inflation at the turn of the decade, financial dollarization remained at high levels throughout the 1990s (see Ize and Levy-Yeyati, 2003 and Oomes (2003) for similar experiences in Mexico, Peru, Uruguay, and Russia). The baseline model suggests that the set of equilibria can change significantly for small changes in policy risk around the thresholds, which might seem to be at odds with this observation. However, the above analysis ignores the fact that citizens might be part of credit chains and thus might also have outstanding claims in both currencies. Here, we present a simple extension in which the buyer is endowed with claims $\left(\hat{b}_f, \hat{b}_l\right)$, with $\hat{b}_c \geqslant 0$, payable to the buyer in the second period. In Appendix C.7, we present a model in which these endowments arise endogenously as a consequence of trading within a credit chain. In this extended setup, the optimal contract solves

$$\max_{b_l, b_f} (1+\lambda) x + \mathbb{E}\left[\theta_b \left(y - \left(\phi_l \left(b_l - \hat{b}_l\right) + \phi_f \left(b_f - \hat{b}_f\right)\right)\right)\right] - x + \mathbb{E}\left[\theta_s \left(y + b_l \phi_l + b_f \phi_f\right)\right]$$

subject to the participation constraint of the seller in (2), the modified participation constraint of the buyer,

$$(1+\lambda) x + \mathbb{E}\left[\theta_b \left(y - \left(\phi_l \left(b_l - \hat{b}_l\right) + \phi_f \left(b_f - \hat{b}_f\right)\right)\right)\right] \geqslant \mathbb{E}\left[\theta_b \left(y + \phi_l \hat{b}_l + \phi_f \hat{b}_f\right)\right],$$

and the payments feasibility constraint

$$\phi_l \left(b_l - \hat{b}_l\right) + \phi_f \left(b_f - \hat{b}_f\right) \leqslant y, \forall \left(\phi_l, \phi_f\right).$$

Note that we no longer impose that the promised payments be non-negative. As we will show, under the following assumption, non-negative payments will indeed be optimal.

Assumption 3. *Assume that*

$$var\left(\theta_s - \theta_b\right) + \lambda \left[var\left(\theta_s\right) - cov\left(\theta_s, \theta_b\right)\right] < \kappa_2,$$

where κ_2 depends on model parameters and is defined in (27).

This assumption requires an upper bound on the variances of taste shocks. The term κ_2 contains a free parameter, $\underline{\phi}_f$, which can be made arbitrarily small in order to satisfy this restriction and Assumption 2. The following proposition shows that hysteresis can be rationalized with our extended model.

Proposition 9. *Under Assumptions 1 and 3, $b_f \geqslant \hat{b}_f$ and $b_l \geqslant \hat{b}_l$.*

The proposition says that even if policy risk is small, the optimal contract will still use a combination of foreign and local currency to denominate contracts (Figure D.3 in the Appendix provides a graphical depiction of this result). In particular, the optimal contract will feature currency matching of stocks, but flows will be denominated in the currency with the largest marginal benefit. Given the presence of positive gains of trade, the buyers will become net debtors in one currency by setting either $b_f > \hat{b}_f$ or $b_l > \hat{b}_l$ to obtain additional x, and pay for it with their endowment of goods y. What this proposition shows is that becoming a net creditor in any currency (i.e., setting $b_c < \hat{b}_c$) is not optimal since it exposes the buyer to an additional source of price risk, which in turn reduces how much the buyer can credibly promise to repay in all states of the world.

To illustrate this result, suppose that θ_s and θ_b are deterministic. Then, we know from previous results that the optimal currency choice only involves comparing price risk across currencies. Notice that with existing obligations, the price level that makes the feasibility constraint bind will now depend on whether $b_c \leqslant \hat{b}_c$ or $b_c > \hat{b}_c$ (in what follows, recall that ϕ_l and ϕ_f are independent). In the former case, the buyer is a net creditor in currency c and higher inflation in currency c is worse for the buyer. Therefore, the relevant price is $\underline{\phi}_c$. In the latter case, the buyer is a net debtor and the relevant price is $\overline{\phi}_c$. The difference in price risk is

$$\frac{\mathbb{E}\left[\phi_l\right]}{\tilde{\phi}_l} - \frac{\mathbb{E}\left[\phi_f\right]}{\tilde{\phi}_f},$$

where $\tilde{\phi}_c \in \left\{\underline{\phi}_c, \overline{\phi}_c\right\}$. Suppose that $b_f < \hat{b}_f$, which implies $b_l > \hat{b}_l$ to satisfy the feasibility constraint with equality. Then, the difference in price risk is

$$\frac{\mathbb{E}\left[\phi_l\right]}{\overline{\phi}_l} - \frac{\mathbb{E}\left[\phi_f\right]}{\underline{\phi}_f} < 0,$$

which implies that increasing b_f and lowering b_l is strictly optimal. Thus, $b_f < \hat{b}_f$ can never be part of an equilibrium contract. A similar argument holds for the local currency. Intuitively, contracts with mismatched positions in currencies are associated with larger price risk, which tightens the payments feasibility constraint and thus lowers the expected amount of goods that can be promised by the buyer. As a result, the optimal contract currency matches stocks (i.e., promises to pay at least the endowment of each currency, $\hat{b}_c$) and denominates flows in the currency with the largest marginal benefit. The proof in the Appendix shows that the above argument generalizes to the case with stochastic taste shocks as long as their variance is not too large. If the variance is very large, then it might be optimal to cannibalize the stocks of foreign currency. This is a situation in which the insurance benefit of the local currency outweighs the price risk consideration behind the provision of the special good.

This result also illustrates why in the baseline model agents would not choose negative promised payments, even if allowed. To see this, note that the contract above is the same contract as in the baseline model without the non-negativity constraints if we redefine promised payments as $\tilde{b}_c = b_c - \hat{b}_c$. Therefore, under Assumption 3, the optimal contract involves $\tilde{b}_c \geqslant 0$.

4 Conclusion

This paper develops a framework to study the optimal choice of currency in the denomination of private contracts in general equilibrium. There are two key channels that determine the optimal currency choice. The first is policy risk stemming from the government's ex-post desire to change the price level, which in turn affects the price risk of denominating contracts in local currency. The second is the covariance between the relative marginal utilities of the agents signing the contract and the price level. The latter channel generates a complementarity between the actions of private agents and those of the government. We show that our model can help explain the cross-country differences in the use of the U.S. dollar to denominate domestic contracts as well as rationalize policy measures aimed at limiting the use of certain currencies.

One advantage of our framework is that its analytical tractability implies that it can be used to study a variety of interesting applications. For example, while we focus on static contracts in our model, it would be interesting to study the interaction between currency choice in long-term contracts and policy. In addition, one could embed this framework in a New Keynesian framework to study the effect of nominal rigidities on the currency choice of contracts. We leave these extensions for future work.

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Appendix: For Online Publication

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A Useful Constants

For the proofs it will be useful to define the following constants, including the term κ_1 in Assumption 2. Define

$$\tilde{\kappa}_1 \equiv \left[(1+\lambda)\,\mathbb{E}\left[\theta_s\right] - \mathbb{E}\left[\theta_b\right]\right] \left(\frac{\mathbb{E}\left[\phi_f\right]}{\overline{\phi}_f} \left(\overline{\theta}_s - \underline{\theta}_b\right) - \left(\mathbb{E}\left[\theta_s\right] - \mathbb{E}\left[\theta_b\right]\right) \right), \tag{17}$$

$$\kappa_1 \equiv \tilde{\kappa}_1 + \frac{1}{2}\left(\mathbb{E}\left[\theta_s\right] - \mathbb{E}\left[\theta_b\right]\right)^2, \tag{18}$$

and

$$\tilde{\kappa}_2 \equiv \operatorname{var}\left(\theta_s - \theta_b\right) + \lambda\left[\operatorname{var}\left(\theta_s\right) - \operatorname{cov}\left(\theta_s, \theta_b\right)\right]. \tag{19}$$

B Omitted Proofs

Proof of Proposition 1

First note that the participation constraint of the seller in problem (4) is binding in the optimum. To see this, suppose it is not binding. Then, increasing x by a little and leaving all remaining variables unchanged is feasible and implies a higher objective function.[17] This

[17] Note that when $\lambda = 0$, the objective function is independent of x. However, it is straightforward to show that the solution (b_l, b_f) to this problem is identical to that when $\lambda > 0$. When $\lambda = 0$, any value of x that satisfies both the seller's and buyer's participation constraints yields the same objective and so we can just focus on the solution in which the seller's participation constraint binds.

implies that at the optimum the participation constraint of the seller is binding. Solving for x using the participation constraint yields the first result of the proposition. Once we substitute the optimal value of x in the problem we obtain the following re-formulated problem:

$$\max_{b_l \geqslant 0, b_f \geqslant 0} \mathbb{E}\left[\left((1+\lambda)\theta_s - \theta_b\right)\left(\phi_l b_l + \phi_f b_f\right)\right]$$

subject to the payments feasibility constraint

$$\overline{\phi}_l b_l + \overline{\phi}_f b_f \leqslant y$$

and the participation constraint of the buyer

$$(1+\lambda)x + \mathbb{E}\left[\theta_b\left(y - b_l\phi_l - b_f\phi_f\right)\right] \geqslant \mathbb{E}\left[\theta_b y\right].$$

This participation constraint will always be slack. To see why, replace the value of x from the binding seller's participation constraint into this constraint and rewrite it to obtain

$$\mathbb{E}\left[\left((1+\lambda)\theta_s - \theta_b\right)\left(b_l\phi_l + b_f\phi_f\right)\right] \geqslant 0,$$

which is identical to the objective function. Consider the contract $b_l = 0, b_f = y/\overline{\phi}_f$. This contract is feasible and Assumption 1 implies that under this contract, the objective function is strictly positive. Consequently, under the optimal contract the objective function will also be strictly positive, implying the slackness of the buyer's participation constraint. Next, we show that the payments feasibility constraint binds. Suppose not. Then increasing b_f by a small amount leaves the constraint unchanged and strictly increases the objective function due to Assumption 1.

Solving for b_f using the payments feasibility constraint and substituting in the objective problem yields the following problem:

$$\max_{b_l \in \left[0, \frac{y}{\overline{\phi}_l}\right]} \mathbb{E}\left[\left((1+\lambda)\theta_s - \theta_b\right)\left(\phi_l b_l + \frac{\phi_f}{\overline{\phi}_f}\left(y - \overline{\phi}_l b_l\right)\right)\right]. \tag{20}$$

The objective is linear in b_l and the derivative with respect to b_l is

$$\mathbb{E}\left[\left(\theta_s(1+\lambda) - \theta_b\right)\left(\phi_l - \frac{\phi_f}{\overline{\phi}_f}\overline{\phi}_l\right)\right].$$

Therefore, the solution is $b_l = y/\overline{\phi}_l$ when the derivative is positive, $b_l = 0$ when the derivative is negative, and any $b_l \in \left[0, y/\overline{\phi}_l\right]$ when the derivative is zero. Q.E.D.

Proof of Proposition 2

The following definitions will be useful for this proof. Define

$$\mathcal{H}(B_l) \equiv (1+\lambda) M_2(B_l) - M_1(B_l),$$

where

$$\begin{aligned} M_2(B_l) &\equiv \mathbb{E}\left[\theta_s\left(\phi_l(B_l) - \frac{\phi_f}{\overline{\phi}_f}\overline{\phi}_l(B_l)\right)\right] \\ &= \mathbb{E}[\theta_s]\,\overline{\hat{\phi}}\left(\frac{\mathbb{E}[\hat{\phi}]}{\overline{\hat{\phi}}} - \frac{\mathbb{E}[\phi_f]}{\overline{\phi}_f}\right) \\ &+ \frac{1}{\psi}\left(\text{var}(\theta_s) - \frac{\mathbb{E}[\phi_f]}{\overline{\phi}_f}\mathbb{E}[\theta_s]\left(\overline{\theta}_s - \underline{\theta}_b\right) - \text{cov}(\theta_s, \theta_b) + \mathbb{E}[\theta_s](\mathbb{E}[\theta_s] - \mathbb{E}[\theta_b])\right) B_l \end{aligned}$$

and

$$\begin{aligned} M_1(B_l) &\equiv \mathbb{E}\left[\theta_b\left(\phi_l(B_l) - \frac{\phi_f}{\overline{\phi}_f}\overline{\phi}_l(B_l)\right)\right] \\ &= \mathbb{E}[\theta_b]\,\overline{\hat{\phi}}\left(\frac{\mathbb{E}[\hat{\phi}]}{\overline{\hat{\phi}}} - \frac{\mathbb{E}[\phi_f]}{\overline{\phi}_f}\right) \\ &- \frac{1}{\psi}\left(\text{var}(\theta_b) + \frac{\mathbb{E}[\phi_f]}{\overline{\phi}_f}\mathbb{E}[\theta_b]\left(\overline{\theta}_s - \underline{\theta}_b\right) - \text{cov}(\theta_s, \theta_b) - \mathbb{E}[\theta_s]\,\mathbb{E}[\theta_b] + \mathbb{E}[\theta_b]^2\right) B_l, \end{aligned}$$

where we have used the best response of the government

$$\phi_l(B_l) = \hat{\phi} + \frac{1}{\psi}(\theta_b - \theta_s) B_l.$$

It will also be useful to compute

$$M_1'(B_l) = -\frac{1}{\psi}\left[\text{var}(\theta_b) + \frac{\mathbb{E}[\phi_f]}{\overline{\phi}_f}\mathbb{E}[\theta_b]\left(\overline{\theta}_s - \underline{\theta}_b\right) - \text{cov}(\theta_s, \theta_b) - \mathbb{E}[\theta_b][\mathbb{E}(\theta_s) - \mathbb{E}[\theta_b]]\right]$$

and

$$M_2'(B_l) = \frac{1}{\psi}\left(\text{var}(\theta_s) - \frac{\mathbb{E}[\phi_f]}{\overline{\phi}_f}\mathbb{E}[\theta_s]\left(\overline{\theta}_s - \underline{\theta}_b\right) - \text{cov}(\theta_s, \theta_b) + \mathbb{E}[\theta_s][\mathbb{E}(\theta_s) - \mathbb{E}[\theta_b]]\right).$$

The function $\mathcal{H}(B_l)$ is useful for characterizing the set of equilibria in this model. This function is obtained by taking the derivative of (20) with respect to b_l and substituting in the government's best response. There are three types of equilibria that can exist. First, an equilibrium with $B_l = 0$ exists if and only if $\mathcal{H}(0) \leqslant 0$. Next, an equilibrium in which $B_f = 0$ can exist if and only if $\mathcal{H}\left(y/\overline{\phi}_l^*\right) \geqslant 0$, where $y/\overline{\phi}_l^*$ corresponds to the maximal feasible value of B_l, and $\overline{\phi}_l^*$ solves

$$\overline{\phi}_l^* = \overline{\hat{\phi}} + \frac{1}{\psi}\left(\overline{\theta}_s - \underline{\theta}_b\right)\frac{y}{\overline{\phi}_l^*}$$

or

$$\overline{\phi}_l^* = \frac{\overline{\hat{\phi}} + \sqrt{\left(\overline{\hat{\phi}}\right)^2 + 4\frac{y}{\psi}\left(\overline{\theta}_s - \underline{\theta}_b\right)}}{2}.$$

Finally, an interior equilibrium exists if and only if there exists some $B_l \in \left(0, y/\overline{\phi}_l^*\right)$ such that $\mathcal{H}(B_l) = 0$.

Define $\mu_1 \equiv \mathbb{E}[\phi_f]/\overline{\phi}_f$. We will show that if $\mathbb{E}\left[\hat{\phi}\right]/\overline{\hat{\phi}} - \mathbb{E}[\phi_f]/\overline{\phi}_f > 0$, then there is a unique equilibrium in which $B_f = 0$ and $B_l = y/\overline{\phi}_l^*$. To see that an equilibrium with $B_l = 0$ cannot exist, notice that

$$\mathcal{H}(0) = \left[(1+\lambda)\,\mathbb{E}[\theta_s] - \mathbb{E}[\theta_b]\right]\overline{\hat{\phi}}\left(\frac{\mathbb{E}\left[\hat{\phi}\right]}{\overline{\hat{\phi}}} - \frac{\mathbb{E}[\phi_f]}{\overline{\phi}_f}\right) > 0.$$

To show that a unique equilibrium with $B_l = y/\overline{\phi}_l^*$ exists, it is sufficient to show that $\mathcal{H}'(B) \geqslant 0$ for all $B \in \left[0, y/\overline{\phi}_l^*\right]$. We have

$$\mathcal{H}'(B) = (1+\lambda)\,M_2'(B) - M_1'(B) = \frac{1}{\psi}\left[\tilde{\kappa}_2 - \tilde{\kappa}_1\right] > 0,$$

where $\tilde{\kappa}_1$ and $\tilde{\kappa}_2$ were defined in (17) and (19) respectively, and it is positive as a consequence of Assumption 2.

Next, define

$$\mu_2 \equiv \frac{\mathbb{E}[\phi_f]}{\overline{\phi}_f} - \frac{1}{\psi}\frac{y}{\overline{\hat{\phi}}\,\overline{\phi}_l^*}\frac{(\tilde{\kappa}_2 - \tilde{\kappa}_1)}{\left[(1+\lambda)\,\mathbb{E}[\theta_s] - \mathbb{E}[\theta_b]\right]}. \tag{21}$$

Notice that Assumptions 1 and 2 implies that $\mu_2 < \mu_1$. We show that for $\mathbb{E}\left[\hat{\phi}\right]/\overline{\hat{\phi}} \in (\mu_2, \mu_1]$, there exist three equilibria. First, we show an equilibrium exists in which $B_l = 0$. We know from above that for this equilibrium to exist it must be that $\mathcal{H}(0) \leqslant 0$. Using the expressions we derived earlier,

$$\mathcal{H}(0) = \left[(1+\lambda)\,\mathbb{E}[\theta_s] - \mathbb{E}[\theta_b]\right]\overline{\hat{\phi}}\left[\frac{\mathbb{E}\left[\hat{\phi}\right]}{\overline{\hat{\phi}}} - \frac{\mathbb{E}[\phi_f]}{\overline{\phi}_f}\right] \leqslant 0,$$

which follows from the case we are considering and Assumption 1. Next, we want to show that there exists an interior equilibrium, i.e. there exists a B such that $\mathcal{H}(B) = 0$. Since we established earlier that $\mathcal{H}'(B) > 0$, there must exist a unique B_l^* such that

$\mathcal{H}\left(B_l^*\right) = 0$. The value B_l^* is

$$B_l^* = \frac{\psi\left[(1+\lambda)\,\mathbb{E}\left[\theta_s\right] - \mathbb{E}\left[\theta_b\right]\right]\overline{\hat{\phi}}\left[\frac{\mathbb{E}[\phi_f]}{\overline{\phi}_f} - \frac{\mathbb{E}[\hat{\phi}]}{\overline{\hat{\phi}}}\right]}{\left[\tilde{\kappa}_2 - \tilde{\kappa}_1\right]}.$$

For this to be strictly interior, a necessary and sufficient condition is

$$B_l^* < \frac{y}{\overline{\phi}_l^*},$$

or

$$\frac{\mathbb{E}\left[\hat{\phi}\right]}{\overline{\hat{\phi}}} > \mu_2.$$

Finally, since $\mathcal{H}'(B) > 0$, it follows that if there is an interior equilibrium, there also must exist an equilibrium with full use of local currency, since it must be that $\mathcal{H}\left(y/\overline{\phi}_l^*\right) > 0$.

Finally, assume that $\mathbb{E}\left[\hat{\phi}\right]/\overline{\hat{\phi}} \leqslant \mu_2$. Given the above analyses, it is straightforward to see that in this case there is a unique equilibrium in which $B_l = 0$. In particular, in this interval, it must be that $\mathcal{H}(B) \leqslant 0$ for all B. Q.E.D.

Proof of Lemma 1

The proof proceeds as follows. We first conjecture that the best response of private agents takes a simple cutoff structure. In particular, we show that if all other agents are playing this cutoff strategy an individual buyer-seller pair also finds it optimal to do so. Finally, we show that such a cutoff strategy is the unique strategy surviving iterated deletion of strictly dominated strategies and characterize it.

We conjecture that the best response takes the following cutoff structure

$$b_l = \begin{cases} 0 & \hat{\xi} < \xi^* \\ \frac{y}{\overline{\phi}_l} & \hat{\xi} \geqslant \xi^* \end{cases}.$$

Suppose that a buyer-seller pair, receiving signal $\hat{\xi}$, believes that all other private agents are following this cutoff strategy. We want to show that the best response of this buyer-seller pair is also a cutoff strategy. Given the signal realization $\hat{\xi}$, this pair believes that the aggregate level of B_l is given by

$$B_l\left(\hat{\xi}\right) = \left[1 - H\left(\xi^* \mid \hat{\xi}\right)\right]\frac{y}{\overline{\phi}_l}$$

where

$$H\left(\xi^* \mid \hat{\xi}\right) \equiv \Pr\left(\xi_j \leqslant \xi^* \mid \hat{\xi}\right)$$

which is the fraction of agents receiving a signal lower than ξ^* conditional on receiving a signal $\hat{\xi}$. Note that

$$\begin{aligned}
H\left(\xi^* \mid \hat{\xi}\right) &= \Pr\left(\varepsilon_j \leqslant \xi^* - \hat{\xi} + \varepsilon_i \mid \hat{\xi}\right) \\
&= \int_{\varepsilon_i} \Pr\left\{\varepsilon_j \leqslant \xi^* - \hat{\xi} + \varepsilon_i \mid \hat{\xi}, \varepsilon_i\right\} \Pr\left(\varepsilon_i \mid \hat{\xi}\right) d\varepsilon_i \\
&= \int_{\varepsilon_i} \Pr\left\{\varepsilon_j \leqslant \xi^* - \hat{\xi} + \varepsilon_i \mid \hat{\xi}, \varepsilon_i\right\} \frac{\Pr\left(\xi = \hat{\xi} - \varepsilon_i\right) \Pr\left(\varepsilon_i\right)}{\int \Pr\left(\xi = \hat{\xi} - \hat{\varepsilon}\right) \Pr\left(\hat{\varepsilon}\right) d\hat{\varepsilon}} d\varepsilon_i \\
&= \int_{\varepsilon_i} \frac{\xi^* - \hat{\xi} + \varepsilon_i + \eta}{2\eta} \frac{1}{2\eta} d\varepsilon_i,
\end{aligned}$$

so that $H\left(\xi^* \mid \hat{\xi}\right)$ is strictly decreasing in $\hat{\xi}$. For future use, it will be useful to note that

$$H\left(\xi^* \mid \xi^*\right) = \int_{\varepsilon_i} \left[\frac{\varepsilon_i}{2\eta} + \frac{1}{2}\right] \frac{1}{2\eta} d\varepsilon_i = \frac{1}{2}.$$

Given $B_l\left(\hat{\xi}\right)$ and the government's best response, we compute the maximal local currency price as follows

$$\overline{\phi}_l\left(\hat{\xi}\right) = \frac{\overline{\hat{\phi}} + \sqrt{\overline{\hat{\phi}}^2 + 4\frac{1}{\psi}\left(\overline{\theta}_s - \underline{\theta}_b\right) H\left(\xi^* \mid \hat{\xi}\right) y}}{2}.$$

Therefore, given signal realization $\hat{\xi}$, the first order condition of the contracting problem with respect to b_l is

$$\left[(1+\lambda)\,\mathbb{E}\left(\theta_s\right) - \mathbb{E}\left(\theta_b\right)\right] \overline{\hat{\phi}} \left(\frac{E\left[\xi \mid \hat{\xi}\right]}{\overline{\hat{\phi}}} - \frac{\mathbb{E}\left(\phi_f\right)}{\overline{\phi}_f}\right) + \frac{1}{\psi}\left[\tilde{\kappa}_2 - \tilde{\kappa}_1\right] B_l\left(\hat{\xi}\right)$$

where $\tilde{\kappa}_1$ and $\tilde{\kappa}_2$ were defined in (17) and (19), respectively. Thus, the optimal choice of b_l satisfies $b_l = 0$ if the above expression is negative, and satisfies $b_l = y/\overline{\phi}_l$ if the above expression is positive.

To characterize the optimal currency choice we need to compute $E\left[\xi \mid \hat{\xi}\right]$. Fix any $\xi \in \left(\underline{\xi}, \overline{\xi}\right)$, then for η small enough, we have that

$$\bar{\xi} - 2\eta > \xi > \underline{\xi} + 2\eta.$$

Therefore,

$$\hat{\xi} - \eta = \xi + \varepsilon_i - \eta \geqslant \xi - 2\eta > \underline{\xi}$$

and

$$\hat{\xi} + \eta = \xi + \varepsilon_i + \eta \leqslant \xi + 2\eta < \overline{\xi}.$$

Then,

$$E\left[\xi \mid \hat{\xi}\right] = \int_{\hat{\xi}-\eta}^{\hat{\xi}+\eta} \xi \Pr\left(\varepsilon = \hat{\xi} - \xi\right) d\xi = \frac{1}{2\eta} \frac{\left(\hat{\xi}+\eta\right)^2 - \left(\hat{\xi}-\eta\right)^2}{2} = \hat{\xi}.$$

Given these computations, define $x\left(\hat{\xi}\right)$ to be the value of x that solves

$$\left[(1+\lambda)\,\mathbb{E}\left(\theta_s\right) - \mathbb{E}\left(\theta_b\right)\right] \overline{\hat{\phi}} \left(\frac{x}{\overline{\overline{\hat{\phi}}}} - \frac{\mathbb{E}\left(\phi_f\right)}{\overline{\phi}_f}\right) + \frac{1}{\psi}\left[\tilde{\kappa}_2 - \tilde{\kappa}_1\right] B_l\left(\hat{\xi}\right) = 0,$$

where $\tilde{\kappa}_1$ and $\tilde{\kappa}_2$ are defined in (17) and (19), respectively. Notice that if there exists a fixed point $x\left(\xi^*\right) = \xi^*$ of the above equation, and the above equation is strictly increasing in $\hat{\xi}$, then the private best response also follows a cutoff strategy with threshold ξ^*. In particular, we will show that the cutoff strategy characterized by ξ^* is the unique strategy surviving iterated deletion of strictly dominated strategies. To do this, we first show that $x\left(\hat{\xi}\right)$ is strictly decreasing. To show this, due to Assumptions 1 and 2, it suffices to show that $B_l'\left(\xi\right) > 0$. We have

$$\begin{aligned}
B_l'\left(\hat{\xi}\right) = &-\frac{H_{\hat{\xi}}\left(\xi^* \mid \hat{\xi}\right) y}{\overline{\phi}_l\left(\hat{\xi}_f\right)} \\
&- \frac{H\left(\xi^* \mid \hat{\xi}\right) y}{\left[\overline{\phi}_l\left(\hat{\xi}_f\right)\right]^2} \frac{1}{2} \left(\left(\overline{\hat{\phi}}^2 + 4\frac{1}{\psi}\left(\overline{\theta}_s - \underline{\theta}_b\right) H\left(\xi^* \mid \hat{\xi}\right) y\right)^{-\frac{1}{2}} \frac{1}{\psi}\left(\overline{\theta}_s - \underline{\theta}_b\right) \left(-H_{\hat{\xi}}\left(\xi^* \mid \hat{\xi}\right)\right)\right) \\
&= -\frac{H_{\hat{\xi}}\left(\xi^* \mid \hat{\xi}\right) y}{\overline{\phi}_l\left(\hat{\xi}_f\right)} \left[1 - \frac{H\left(\xi^* \mid \hat{\xi}\right) y}{\overline{\phi}_l\left(\hat{\xi}_f\right)} \frac{1}{2} \left(\left(\overline{\hat{\phi}}^2 + 4\frac{1}{\psi}\left(\overline{\theta}_s - \underline{\theta}_b\right) H\left(\xi^* \mid \hat{\xi}\right) y\right)^{-\frac{1}{2}} \frac{1}{\psi}\left(\overline{\theta}_s - \underline{\theta}_b\right)\right)\right].
\end{aligned}$$

Let us consider the term in square brackets. Since we have already established that $H_{\hat{\xi}}\left(\xi^* \mid \hat{\xi}\right) < 0$, we want to show that

$$1 - H\left(\xi^* \mid \hat{\xi}\right) \frac{y}{\overline{\phi}_l\left(\hat{\xi}_f\right)} \frac{1}{2} \left(\left(\overline{\hat{\phi}}^2 + 4\frac{1}{2\psi}\left(\overline{\theta}_s - \underline{\theta}_b\right) H\left(\xi^* \mid \hat{\xi}\right) y\right)^{-\frac{1}{2}} \frac{1}{\psi}\left(\overline{\theta}_s - \underline{\theta}_b\right)\right) > 0.$$

A sufficient condition for this is

$$\begin{aligned}
&\sqrt{\overline{\hat{\phi}}^2 + 4\frac{1}{\psi}\left(\overline{\theta}_s - \underline{\theta}_b\right) H\left(\xi^* \mid \hat{\xi}\right) y} \\
&> H\left(\xi^* \mid \hat{\xi}\right) y \left(\left(\overline{\hat{\phi}}^2 + 4\frac{1}{\psi}\left(\overline{\theta}_s - \underline{\theta}_b\right) H\left(\xi^* \mid \hat{\xi}\right) y\right)^{-\frac{1}{2}} \frac{1}{\psi}\left(\overline{\theta}_s - \underline{\theta}_b\right)\right)
\end{aligned}$$

or

$$\overline{\hat{\phi}}^2 + \frac{1}{\psi}\left(\overline{\theta}_s - \underline{\theta}_b\right) H\left(\xi^* \mid \hat{\xi}\right) y > 0,$$

which is true. Therefore, $B_l'\left(\hat{\xi}\right) > 0$ and so $x\left(\hat{\xi}\right)$ is strictly decreasing.

We will next show that if a strategy b_l survives n rounds of iterated deletion of strictly dominated strategies, then

$$b_l(\xi) = \begin{cases} 0 & \xi < x^{n-1}(\mu_2) \\ \frac{y}{\overline{\phi}_l} & \xi > x^{n-1}(\mu_1) \end{cases},$$

where μ_1 and μ_2 were defined in Proposition 2. It is easy to see that this claim is true for $n = 1$ since

$$b_l(\xi) = \begin{cases} 0 & \xi < x^0(\mu_2) = \mu_2 \\ \frac{y}{\overline{\phi}_l} & \xi > x^0(\mu_1) = \mu_1 \end{cases},$$

which follows from the definitions of μ_1 and μ_2. Now suppose the claim is true for some $n > 1$. Then, if a particular buyer-seller pair knew that all other pairs choose $b_l = 0$ if $\xi < x^{n-1}(\mu_2)$ and $b_l = \frac{y}{\overline{\phi}_l}$ if $\xi > x^{n-1}(\mu_1)$, its best response would be to choose $b_l = 0$ if the signal was below $x\left(x^{n-1}(\mu_2)\right)$. Since $x(\cdot)$ is strictly decreasing, it has a unique fixed point satisfying

$$[(1+\lambda)\mathbb{E}(\theta_s) - \mathbb{E}(\theta_b)]\,\overline{\hat{\phi}}\left(\frac{\xi^*}{\overline{\hat{\phi}}} - \frac{\mathbb{E}(\phi_f)}{\overline{\phi}_f}\right) + \frac{1}{\psi}[\tilde{\kappa}_2 - \tilde{\kappa}_1]\,B_l(\xi^*) = 0$$

and $x^n(\mu_2) \to \xi^*$ as $n \to \infty$. An identical argument holds for $x^n(\mu_1)$.

We can now solve for the fixed point

$$[(1+\lambda)\mathbb{E}(\theta_s) - \mathbb{E}(\theta_b)]\,\overline{\hat{\phi}}\left(\frac{\xi^*}{\overline{\hat{\phi}}} - \frac{\mathbb{E}(\phi_f)}{\overline{\phi}_f}\right) + \frac{1}{\psi}[\tilde{\kappa}_2 - \tilde{\kappa}_1]\,\frac{1}{2}\frac{y}{\overline{\phi}_l^{**}} = 0,$$

where

$$\overline{\phi}_l^{**} \equiv \frac{\overline{\hat{\phi}} + \sqrt{\overline{\hat{\phi}}^2 + 4\frac{1}{\psi}\left(\overline{\theta}_s - \underline{\theta}_b\right)\frac{1}{2}y}}{2}. \tag{22}$$

is the maximal price level arising in this equilibrium. Therefore,

$$\xi^* = \overline{\hat{\phi}}\frac{\mathbb{E}(\phi_f)}{\overline{\phi}_f} - \frac{1}{2}\frac{1}{\psi}\frac{(\tilde{\kappa}_2 - \tilde{\kappa}_1)}{[(1+\lambda)\mathbb{E}[\theta_s] - \mathbb{E}[\theta_b]]}\frac{y}{\overline{\phi}_l^{**}}. \tag{23}$$

and so in equilibrium, we have that

$$b_l\left(\hat{\xi}\right) = \begin{cases} 0 & \hat{\xi} < \xi^* \\ \frac{y}{\overline{\phi}_l^{**}} & \hat{\xi} \geqslant \xi^* \end{cases}.$$

This completes the proof. Q.E.D.

Proof of Proposition 3

The proof of the first part follows directly from Lemma 1. Define $\mu_{GG} \equiv \xi^*/\overline{\hat{\phi}}$ where ξ^* was defined in (23). As $\eta \to 0$, we have

$$b_l\left(\xi\right) = \begin{cases} 0 & \xi < \xi^* \\ \frac{y}{\overline{\phi}_l^{**}} & \xi \geqslant \xi^* \end{cases}$$

and thus, it follows that $B_l = y/\overline{\phi}_l^{**}$ if $\mathbb{E}\left[\hat{\phi}\right]/\overline{\hat{\phi}} > \mu_{GG}$ and $B_l = 0$ otherwise. Next, notice that

$$\mu_1 - \mu_{GG} = \frac{1}{2}\frac{1}{\psi}\frac{\left(\tilde{\kappa}_2 - \tilde{\kappa}_1\right)}{\left[\left(1+\lambda\right)\mathbb{E}\left[\theta_s\right] - \mathbb{E}\left[\theta_b\right]\right]}\frac{y}{\overline{\phi}_l^{**}} > 0$$

since $\tilde{\kappa}_2 - \tilde{\kappa}_1 > 0$ due to Assumption 2 and

$$\mu_{GG} - \mu_2 = \frac{\left(\tilde{\kappa}_2 - \tilde{\kappa}_1\right)}{\left[\left(1+\lambda\right)\mathbb{E}\left[\theta_s\right] - \mathbb{E}\left[\theta_b\right]\right]}\frac{1}{\psi}\frac{y}{\overline{\hat{\phi}}}\left[\frac{1}{\overline{\phi}_l^{*}} - \frac{1}{2\overline{\phi}_l^{**}}\right] > 0$$

which follows from Assumption 2 and the fact that

$$2\overline{\phi}_l^{**} - \overline{\phi}_l^{*} = \frac{1}{2}\left[\overline{\hat{\phi}} + 2\sqrt{\left(\overline{\hat{\phi}}\right)^2 + 2\frac{1}{\psi}\left(\overline{\theta}_s - \underline{\theta}_b\right)y} - \sqrt{\left(\overline{\hat{\phi}}\right)^2 + 4\frac{y}{\psi}\left(\overline{\theta}_s - \underline{\theta}_b\right)}\right] > 0.$$

Therefore, $\mu_2 < \mu_{GG} < \mu_1$. Q.E.D.

Proof of Proposition 4

Since the planner's objective is increasing in x, the participation constraint of the seller and the payments feasibility constraint will bind, which allows us to substitute for x in the objective function. Thus, we can write the planner's problem as

$$\max_{B_l}\mathbb{E}\left(\left[\left(1+\lambda\right)\theta_s - \theta_b\right]\left(\left(\phi_l - \frac{\phi_f}{\overline{\overline{\phi}}_f}\overline{\phi}_l\right)B_l + \frac{\phi_f}{\overline{\overline{\phi}}_f}y\right) + \left(\theta_s + \theta_b\right)y - l\left(\phi_l\right)\right),$$

subject to the buyer's participation constraint (3), and the government's optimal policy in (8) and (9). An identical argument to the one used in the characterization of the competitive equilibria implies that the participation constraint of the buyer will be slack. Given our previous definitions, it will be useful to define the planning problem as follows:

$$SP(B) \equiv \max_{B} \left[(1+\lambda) M_2(B) B - M_1(B) B - \mathbb{E} l(\phi_l(B))\right] + \tilde{y},$$

where $\tilde{y} \equiv \left(\mathbb{E}[\theta_s] + \mathbb{E}[\theta_b] + [(1+\lambda)\mathbb{E}[\theta_s] - \mathbb{E}[\theta_b]]\, \mathbb{E}[\phi_f] / \overline{\phi}_f\right) y$, subject to

$$\phi_l(B) = \hat{\phi} + \frac{1}{\psi}(\theta_s - \theta_b) B.$$

The derivative of the objective function is

$$\begin{aligned} SP'(B) &= \left[(1+\lambda)\left[M_2(B) + M_2'(B) B\right] - M_1(B) - M_1'(B) B - \mathbb{E} l'(\phi_l(B))\, \phi_l'(B)\right] \\ &= \left[(1+\lambda) M_2(B) - M_1(B) + \Delta(B) B\right], \end{aligned}$$

where we have used the definition of $l(\phi)$ and

$$\Delta(B) \equiv (1+\lambda) M_2'(B) - M_1'(B) - \mathbb{E}(\theta_s - \theta_b)\, \phi_l'(B).$$

Next, let us check the second derivative of the planner's objective function. First, we have

$$\Delta'(B) = (1+\lambda) M_2''(B) - M_1''(B) - \mathbb{E}(\theta_s - \theta_b)\, \phi_l''(B) = 0,$$

which implies that

$$\begin{aligned} SP''(B) &= (1+\lambda) M_2'(B) - M_1'(B) + \Delta(B) \\ &= 2(1+\lambda) M_2'(B) - 2M_1'(B) - \mathbb{E}(\theta_s - \theta_b)\, \phi_l'(B) \\ &= \frac{2}{\psi}\left(\frac{1}{2}\text{var}(\theta_s - \theta_b) + \lambda\left[\text{var}(\theta_s) - \text{cov}(\theta_s, \theta_b)\right] - \kappa_1\right) \\ &> 0, \end{aligned}$$

where κ_1 was defined in (18), and where the last inequality follows from Assumption 2. Therefore, the planner's problem is strictly convex, which implies that computing the solution involves comparing the value of the objective at end points $B_l = 0$ and $B_l = y/\overline{\phi}_l^*$. Note that the maximal feasible level of B_l depends only on parameters and thus is identical across both the competitive equilibrium and the planner's problem.

Define

$$\mu_{SP} \equiv \frac{\mathbb{E}[\phi_f]}{\overline{\phi}_f} - \frac{1}{\psi}\frac{y}{\overline{\hat{\phi}}\overline{\phi}_l^*}\frac{(\tilde{\kappa}_2 - \tilde{\kappa}_1) - \frac{1}{2}\mathbb{E}\left((\theta_s - \theta_b)^2\right)}{[(1+\lambda)\,\mathbb{E}[\theta_s] - \mathbb{E}[\theta_b]]}.$$

We have

$$SP(0) = \tilde{y}$$

and

$$SP\left(\frac{y}{\overline{\phi}_l^*}\right) = \tilde{y} + (1+\lambda)\,M_2\left(\frac{y}{\overline{\phi}_l^*}\right)\frac{y}{\overline{\phi}_l^*} - M_1\left(\frac{y}{\overline{\phi}_l^*}\right)\frac{y}{\overline{\phi}_l^*} - \frac{\psi}{2}\mathbb{E}\left(\frac{1}{\psi}(\theta_s - \theta_b)\frac{y}{\overline{\phi}_l^*}\right)^2.$$

Thus, to compare the above two terms, we need to compute the sign of

$$\begin{aligned}
&(1+\lambda)\,M_2\left(\frac{y}{\overline{\phi}_l^*}\right)\frac{y}{\overline{\phi}_l^*} - M_1\left(\frac{y}{\overline{\phi}_l^*}\right)\frac{y}{\overline{\phi}_l^*} - \frac{\psi}{2}\mathbb{E}\left(\frac{1}{\psi}(\theta_s - \theta_b)\frac{y}{\overline{\phi}_l^*}\right)^2 \\
&= \left[\left(\frac{\mathbb{E}\left[\hat{\phi}\right]}{\overline{\hat{\phi}}} - \frac{\mathbb{E}[\phi_f]}{\overline{\phi}_f}\right) + \frac{1}{\psi}\frac{y}{\overline{\hat{\phi}}\overline{\phi}_l^*}\frac{(\tilde{\kappa}_2 - \tilde{\kappa}_1) - \frac{1}{2}\mathbb{E}\left((\theta_s - \theta_b)^2\right)}{[(1+\lambda)\,\mathbb{E}[\theta_s] - \mathbb{E}[\theta_b]]}\right]\frac{y}{\overline{\phi}_l^*},
\end{aligned}$$

which immediately implies the result given threshold μ_{SP}. Finally, it is easy to see that $\mu_{SP} < \mu_1$ and a simple computation implies that

$$\mu_{SP} - \mu_2 = \frac{1}{2\psi}\frac{\mathbb{E}\left((\theta_s - \theta_b)^2\right)}{((1+\lambda)\,\mathbb{E}(\theta_s) - \mathbb{E}(\theta_b))}\frac{y}{\overline{\phi}_l^*} > 0,$$

which proves that $\mu_2 < \mu_{SP} < \mu_1$. Q.E.D.

Proof of Proposition 5

Given the definitions of ξ^* and μ_{SP}, we have (recall that $\mu_{GG} = \xi^*/\overline{\hat{\phi}}$)

$$\begin{aligned}
&\mu_{GG} - \mu_{SP} \\
=&\frac{(\tilde{\kappa}_2 - \tilde{\kappa}_1)}{[(1+\lambda)\,\mathbb{E}[\theta_s] - \mathbb{E}[\theta_b]]}\frac{1}{\psi}\frac{y}{\overline{\hat{\phi}}}\left[\frac{1}{\overline{\phi}_l^*} - \frac{1}{2}\frac{1}{\overline{\phi}_l^{**}}\right] - \frac{1}{\psi}\frac{y}{\overline{\hat{\phi}}\overline{\phi}_l^*}\left(\frac{\mathbb{E}\left((\theta_s - \theta_b)^2\right)}{2\,((1+\lambda)\,\mathbb{E}(\theta_s) - \mathbb{E}(\theta_b))}\right)
\end{aligned}$$

where $\tilde{\kappa}_2$ and $\tilde{\kappa}_1$ were defined in (19) and (17), respectively. Define

$$\iota \equiv 1 - \frac{1}{2}\frac{\overline{\phi}_l^*}{\overline{\phi}_l^{**}}.$$

We know from the proof of Proposition 3 that $\iota > 0$. Then we have

$$\mu_{GG} - \mu_{SP} = \zeta\Delta$$

where

$$\zeta \equiv \frac{1}{[(1+\lambda)\,\mathbb{E}\,[\theta_s] - \mathbb{E}\,[\theta_b]]}\frac{1}{\psi}\frac{y}{\hat{\overline{\phi}}\overline{\phi}_l^*} > 0$$

and

$$\Delta \equiv \iota\,(\tilde{\kappa}_2 - \tilde{\kappa}_1) - \frac{1}{2}\mathbb{E}\left((\theta_s - \theta_b)^2\right).$$

Therefore $\mu_{GG} - \mu_{SP} > 0$ if and only if

$$\iota\,(\tilde{\kappa}_2 - \tilde{\kappa}_1) - \frac{1}{2}\mathbb{E}\left((\theta_s - \theta_b)^2\right) > 0,$$

which is the condition in the Proposition. Thus, if $\Delta > 0$ (< 0) then $\mu_{GG} > \mu_{SP}$ ($\mu_{GG} < \mu_{SP}$). Q.E.D.

Proof of Proposition 6

Before proving the proposition it will be useful to compute the following:

$$\mathbb{E}\,[\theta_s] = \int_{\underline{\chi}}^{1} 0 dF(\chi) + \int_{1}^{\bar{\chi}} 1 dF(\chi) = (1 - F(1))$$

$$\text{var}\,(\theta_s) = (1 - F(1))\,F(1)$$

$$\mathbb{E}\,[\theta_b] = \int_{\underline{\chi}}^{1} \chi dF(\chi) + \int_{1}^{\bar{\chi}} 1 dF(\chi) = F(1)\,\mathbb{E}\,[\chi \mid \chi \leqslant 1] + (1 - F(1))$$

$$\mathbb{E}\,[\theta_b\theta_s] = \int_{1}^{\bar{\chi}} 1 dF(\chi) = (1 - F(1))$$

$$\mathbb{E}\,[\theta_s - \theta_b] = -\int_{\underline{\chi}}^{1} \chi dF(\chi) = -F(1)\,\mathbb{E}\,[\chi \mid \chi \leqslant 1]$$

$$\mathbb{E}\left[(\theta_s - \theta_b)^2\right] = \int_{\underline{\chi}}^{1} \chi^2 dF(\chi) = F(1)\,\mathbb{E}\left[\chi^2 \mid \chi \leqslant 1\right]$$

$$\text{var}(\theta_s - \theta_b) = F(1)\,\mathbb{E}\left[\chi^2 \mid \chi \leqslant 1\right] - \left(F(1)\,\mathbb{E}[\chi \mid \chi \leqslant 1]\right)^2$$

$$\text{cov}(\theta_s, \theta_b) = \mathbb{E}[\theta_b\theta_s] - \mathbb{E}[\theta_b]\,\mathbb{E}[\theta_s] = (1 - F(1))\,F(1)\,(1 - \mathbb{E}[\chi \mid \chi \leqslant 1])$$

Proof of Proposition 6.

Consider the contracting problem for the model with default. First notice that the non-negativity constraint on consumption is equivalent to imposing the payments feasibility constraint in those states of the world in which the buyer repays. By a similar argument used in the proof of Proposition 1, the participation constraint of the seller will be binding and the participation constrain of the buyer will be slack. Substituting the participation constraint of the seller and payments feasibility constraint (in the no-default states) into the objective yields

$$\max_{b_l \in \left[0, \frac{y}{\phi_l}\right]} \lambda\mathbb{E}\left[y + \left(\phi_l b_l + \frac{\phi_f}{\overline{\phi}_f}\left(y - \overline{\phi}_l b_l\right)\right)\mathbb{I}_{\chi \geqslant 1}\right] + \mathbb{E}\left[y - \chi\left(\phi_l b_l + \frac{\phi_f}{\overline{\phi}_f}\left(y - \overline{\phi}_l b_l\right)\right)\mathbb{I}_{\chi < 1}\right].$$

Next, consider the problem with taste shocks in (20). Using the mapping of taste shocks in (12) and (13), the problem becomes

$$\max_{b_l \in \left[0, \frac{y}{\phi_l}\right]} \lambda\mathbb{E}\left[y + \left(\phi_l b_l + \frac{\phi_f}{\overline{\phi}_f}\left(y - \overline{\phi}_l b_l\right)\right)\mathbb{I}_{\chi \geqslant 1}\right] + \mathbb{E}\left[\chi y - \chi\left(\phi_l b_l + \frac{\phi_f}{\overline{\phi}_f}\left(y - \overline{\phi}_l b_l\right)\right)\mathbb{I}_{\chi < 1}\right],$$

so that the two problems only differ by a constant. Thus, they have the same solution. It is also easy to see that the problems for the government coincide. Thus, the set of equilibrium outcomes is identical.

Next, we show that the implied taste shock model satisfies Assumptions 1 and 2. Using the calculations prior to this proof we have

$$\begin{aligned}(1+\lambda)\,\mathbb{E}[\theta_s] - \mathbb{E}[\theta_b] &= (1+\lambda)(1 - F_\chi(1)) - F_\chi(1)\,\mathbb{E}[\chi \mid \chi \leqslant 1] - (1 - F_\chi(1)) \\ &= \lambda(1 - F_\chi(1)) - F_\chi(1)\,\mathbb{E}[\chi \mid \chi \leqslant 1],\end{aligned}$$

which is strictly positive as a consequence of the assumption in the proposition. Next, let us verify that Assumption 2 is satisfied. Notice that the term κ_1 can be written as

$$\begin{aligned}\kappa_1 &= \left[(1+\lambda)\,\mathbb{E}\left[\theta_s\right]-\mathbb{E}\left[\theta_b\right]\right]\left(\frac{\mathbb{E}\left[\phi_f\right]}{\overline{\phi}_f}\left(\overline{\theta}_s-\underline{\theta}_b\right)-\left(\mathbb{E}\left[\theta_s\right]-\mathbb{E}\left[\theta_b\right]\right)\right)+\frac{1}{2}\left(\mathbb{E}\left[\theta_s\right]-\mathbb{E}\left[\theta_b\right]\right)^2\\ &\leqslant -\left[(1+\lambda)\,\mathbb{E}\left[\theta_s\right]-\mathbb{E}\left[\theta_b\right]\right]\left(\mathbb{E}\left[\theta_s\right]-\mathbb{E}\left[\theta_b\right]\right)+\frac{1}{2}\left(\mathbb{E}\left[\theta_s\right]-\mathbb{E}\left[\theta_b\right]\right)^2\end{aligned}$$

since $\overline{\theta}_s-\underline{\theta}_b=0$ if $\chi\geqslant 1$ and $\overline{\theta}_s-\underline{\theta}_b=-\chi$ if $\chi\leqslant 1$. We have

$$\begin{aligned}&\frac{1}{2}\mathrm{var}\left(\theta_s-\theta_b\right)+\lambda\left[\mathrm{var}\left(\theta_s\right)-\mathrm{cov}\left(\theta_s,\theta_b\right)\right]\\ =&\frac{1}{2}\left[F(1)\,\mathbb{E}\left[\chi^2\mid\chi\leqslant 1\right]-\left(F(1)\,\mathbb{E}\left[\chi\mid\chi\leqslant 1\right]\right)^2\right]+\lambda\left(1-F(1)\right)F(1)\left[\mathbb{E}\left[\chi\mid\chi\leqslant 1\right]\right]\end{aligned}$$

and

$$\begin{aligned}&-\left[(1+\lambda)\,\mathbb{E}\left[\theta_s\right]-\mathbb{E}\left[\theta_b\right]\right]\left(\mathbb{E}\left[\theta_s\right]-\mathbb{E}\left[\theta_b\right]\right)+\frac{1}{2}\left(\mathbb{E}\left[\theta_s\right]-\mathbb{E}\left[\theta_b\right]\right)^2\\ =&\left[\lambda\left(1-F(1)\right)-F(1)\,\mathbb{E}\left[\chi\mid\chi\leqslant 1\right]\right]F(1)\,\mathbb{E}\left[\chi\mid\chi\leqslant 1\right]+\frac{1}{2}\left(F(1)\,\mathbb{E}\left[\chi\mid\chi\leqslant 1\right]\right)^2.\end{aligned}$$

Therefore,

$$\begin{aligned}&\frac{1}{2}\mathrm{var}\left(\theta_s-\theta_b\right)+\lambda\left[\mathrm{var}\left(\theta_s\right)-\mathrm{cov}\left(\theta_s,\theta_b\right)\right]\\ &+\left[(1+\lambda)\,\mathbb{E}\left[\theta_s\right]-\mathbb{E}\left[\theta_b\right]\right]\left(\mathbb{E}\left[\theta_s\right]-\mathbb{E}\left[\theta_b\right]\right)-\frac{1}{2}\left(\mathbb{E}\left[\theta_s\right]-\mathbb{E}\left[\theta_b\right]\right)^2\\ =&\frac{1}{2}F(1)\,\mathbb{E}\left[\chi^2\mid\chi\leqslant 1\right]\\ >&0,\end{aligned}$$

which proves the result.

Proof of Proposition 7

The proof of this proposition requires the following lemma.

Lemma 2. *In the optimal international bilateral contract, the amount of special good is given by*

$$\tilde{x}_i=\mathbb{E}\left[\theta_{js}\left(\phi_i\tilde{b}_{ii}+\phi_j\tilde{b}_{ij}+\phi_f\tilde{b}_{if}\right)\right].$$

Additionally, for any currency c*, the optimal payments are given by* $\tilde{b}_{ic}=\upsilon_c y/\overline{\phi}_c$ *with* $\upsilon_c\in[0,1]$*,*

$\sum_{k=i,j,f} \upsilon_k = 1$, *and* $\upsilon_c = 0$ *if*

$$\mathbb{E}\left[\left((1+\lambda)\,\theta_{js} - \theta_{ib}\right)\left(\frac{\phi_c}{\overline{\phi}_c}\right)\right] < \max_{k=i,j,f} \mathbb{E}\left[\left((1+\lambda)\,\theta_{js} - \theta_{ib}\right)\left(\frac{\phi_k}{\overline{\phi}_k}\right)\right].$$

Proof. We can use the same argument used in the baseline model to show that the participation constraint of the seller in problem (14) is binding and the participation constraint of the buyer is slack at the optimum. We then solve for $\tilde{x}_i$ using the participation constraint of the seller. Once we substitute $\tilde{x}_i$ in the problem, we obtain the following re-formulated problem:

$$\max_{\tilde{b}_{ii} \geqslant 0, \tilde{b}_{ij} \geqslant 0, \tilde{b}_{if} \geqslant 0} \mathbb{E}\left[\left((1+\lambda)\,\theta_{js} - \theta_{ib}\right)\left(\phi_i \tilde{b}_{ii} + \phi_j \tilde{b}_{ij} + \phi_f \tilde{b}_{if}\right)\right]$$

subject to the feasibility constraint

$$\overline{\phi}_i \tilde{b}_{ii} + \overline{\phi}_j \tilde{b}_{ij} + \overline{\phi}_f \tilde{b}_{if} \leqslant y.$$

This is a linear problem whose solution involves corners. We solve this by supposing $b_c = 0$ and then the problem is the same as (4), which we solve using Proposition 1. We do this for $c = i, j, f$ and then compare the objective function in each of the three cases. Comparing the values yields the results stated in the proposition. Q.E.D.

Proof of Proposition 7.

Suppose that $\gamma = 1$. We restrict attention to symmetric equilibria in which $\tilde{B}_{jc} = \tilde{B}_{ic} \equiv \tilde{B}_c$ for $c = i, j, f$. The proof of the proposition proceeds in two steps. First, we compute a threshold for policy risk below which there is an equilibrium in which $\tilde{B}_i = 0$, $\tilde{B}_j = 0$ and $\tilde{B}_f = y/\overline{\phi}_f$. We next find the threshold below which the equilibrium is unique.

In order for $\tilde{B}_i = 0$, $\tilde{B}_j = 0$, and $\tilde{B}_f = y/\overline{\phi}_f$ to be an equilibrium, the marginal value of signing the contract in currency f has to be larger than the marginal values of doing it in currency i and j:

$$\frac{\mathbb{E}\left[\left(\theta_{js}\,(1+\lambda) - \theta_{ib}\right)\phi_f\right]}{\overline{\phi}_f} > \frac{\mathbb{E}\left[\left(\theta_{js}\,(1+\lambda) - \theta_{ib}\right)\phi_i\right]}{\overline{\phi}_i} \tag{24}$$

and

$$\frac{\mathbb{E}\left[\left(\theta_{js}\,(1+\lambda) - \theta_{ib}\right)\phi_f\right]}{\overline{\phi}_f} > \frac{\mathbb{E}\left[\left(\theta_{js}\,(1+\lambda) - \theta_{ib}\right)\phi_j\right]}{\overline{\phi}_j}. \tag{25}$$

These conditions ensure that contracts between buyers from country i and sellers from country j are set in currency f. We also need conditions for which contracts between buyers from country j and sellers from country i are set in currency f, but these are equivalent to the previous ones given the symmetry across countries. After substituting in the governments' best responses and evaluating these expressions at $\tilde{B}_i = 0$, $\tilde{B}_j = 0$ and $\tilde{B}_f = y/\overline{\phi}_f$, these optimality conditions simplify to $\mu_1 = \mathbb{E}[\phi_f]/\overline{\phi}_f > \mathbb{E}[\hat{\phi}_i]/\overline{\hat{\phi}}_i = \mathbb{E}[\hat{\phi}_j]/\overline{\hat{\phi}}_j$. These are identical to the conditions obtained in the baseline model.

Now we show the conditions under which this equilibrium is unique in the set of symmetric equilibria. For this to be a unique equilibrium, it must also be true that the above inequalities hold for all prices ϕ_i consistent with $\tilde{B}_i \in \left[0, y/\overline{\phi}_i^*\right]$. Note that imposing symmetry in the currency choices of international contracts yields the following optimal choice of inflation for the government of country i

$$\phi_i = \hat{\phi}_i + \frac{1}{\psi}\left(\theta_{is} - \theta_{ib}\right)\tilde{B}_i.$$

Additionally, the minimum level of inflation (maximum level of ϕ) is the same as in the baseline economy: $\overline{\phi}_i = \overline{\hat{\phi}}_i + \frac{1}{\psi}\left(\overline{\theta} - \underline{\theta}\right)\tilde{B}_i$. We obtain symmetric expressions for ϕ_j. Replacing the government's choice of inflation in inequality (24) yields

$$\frac{\mathbb{E}\left[\left(\theta_{js}\left(1+\lambda\right) - \theta_{ib}\right)\phi_f\right]}{\overline{\phi}_f} > \frac{\mathbb{E}\left[\left(\theta_{js}\left(1+\lambda\right) - \theta_{ib}\right)\left(\hat{\phi}_i + \frac{1}{\psi}\left(\theta_{is} - \theta_{ib}\right)\tilde{B}_i\right)\right]}{\overline{\hat{\phi}}_i + \frac{1}{\psi}\left(\overline{\theta}_s - \underline{\theta}_b\right)\tilde{B}_i}$$

or equivalently

$$\left(\left(1+\lambda\right)\mathbb{E}\left[\theta_s\right] - \mathbb{E}\left[\theta_b\right]\right)\overline{\hat{\phi}}_i\left(\frac{\mathbb{E}\left[\phi_f\right]}{\overline{\phi}_f} - \frac{\mathbb{E}\left[\hat{\phi}_i\right]}{\overline{\hat{\phi}}_i}\right) > \frac{1}{\psi}\left(\mathrm{var}\left(\theta_b\right) - \mathrm{cov}\left(\theta_b, \theta_s\right) - \tilde{\kappa}_1\right)\tilde{B}_i.$$

To check if this inequality holds for all $\tilde{B}_i$ we need to sign the expression in parentheses on the right side of the above expression. If it is negative then we know this holds for all $\tilde{B}_i$ since $\mathbb{E}[\hat{\phi}_i]/\overline{\hat{\phi}}_i < \mathbb{E}[\phi_f]/\overline{\phi}_f$. If it is positive, a necessary condition to have a unique foreign currency equilibrium is

$$\frac{\mathbb{E}\left[\hat{\phi}_i\right]}{\overline{\hat{\phi}}_i} < \frac{\mathbb{E}\left(\phi_f\right)}{\overline{\phi}_f} - \frac{1}{\psi\overline{\hat{\phi}}_i}\left(\frac{\mathrm{var}\left(\theta_b\right) - \mathrm{cov}\left(\theta_b, \theta_s\right) - \tilde{\kappa}_1}{\left(\left(1+\lambda\right)\mathbb{E}\left(\theta_s\right) - \mathbb{E}\left(\theta_b\right)\right)}\right)\frac{y}{\overline{\phi}_i^*}$$

Replacing the government's choice of inflation in inequality (25) yields

$$\frac{\mathbb{E}\left[\left(\theta_{js}\left(1+\lambda\right)-\theta_{ib}\right)\phi_{f}\right]}{\overline{\phi}_{f}}>\frac{\mathbb{E}\left[\left(\theta_{js}\left(1+\lambda\right)-\theta_{ib}\right)\left(\hat{\phi}_{j}+\frac{1}{\psi}\left(\theta_{js}-\theta_{jb}\right)\tilde{B}_{j}\right)\right]}{\overline{\hat{\phi}}_{j}+\frac{1}{\psi}\left(\overline{\theta}_{s}-\underline{\theta}_{b}\right)\tilde{B}_{j}}$$

or

$$\left(\frac{\mathbb{E}\left[\phi_{f}\right]}{\overline{\phi}_{f}}-\frac{\mathbb{E}\left[\hat{\phi}_{j}\right]}{\overline{\hat{\phi}}_{j}}\right)>\frac{1}{\psi\overline{\hat{\phi}}_{j}}\left(\frac{\left(1+\lambda\right)\left(\operatorname{var}\left(\theta_{s}\right)-\operatorname{cov}\left(\theta_{b},\theta_{s}\right)\right)-\tilde{\kappa}_{1}}{\left(\mathbb{E}\left[\theta_{s}\right]\left(1+\lambda\right)-\mathbb{E}\left[\theta_{b}\right]\right)}\right)\tilde{B}_{j}.$$

As before, we need to sign the expression on the right hand side. If it is negative then we know this holds for all $\tilde{B}_j$. If it is positive then we need

$$\frac{\mathbb{E}\left[\hat{\phi}_{j}\right]}{\overline{\hat{\phi}}_{j}}<\frac{\mathbb{E}\left[\phi_{f}\right]}{\overline{\phi}_{f}}-\frac{1}{\psi\overline{\hat{\phi}}_{j}}\left(\frac{\left(1+\lambda\right)\left(\operatorname{var}\left(\theta_{s}\right)-\operatorname{cov}\left(\theta_{b},\theta_{s}\right)\right)-\tilde{\kappa}_{1}}{\left(\mathbb{E}\left[\theta_{s}\right]\left(1+\lambda\right)-\mathbb{E}\left[\theta_{b}\right]\right)}\right)\frac{y}{\overline{\phi}_{j}^{*}}.$$

Given assumptions, we have $\overline{\phi}_{i}^{*}=\overline{\phi}_{j}^{*}=\overline{\phi}_{l}^{*}$. Since both inequalities need to hold simultaneously, the cutoff value of policy risk below which the equilibrium with $\tilde{B}_i=0$, $\tilde{B}_j=0$ and $\tilde{B}_f=y/\overline{\phi}_f$ is the unique symmetric equilibrium is given by

$$\mu_{2}^{I}=\min\left\{\frac{\mathbb{E}\left[\phi_{f}\right]}{\overline{\phi}_{f}},\frac{\mathbb{E}\left[\phi_{f}\right]}{\overline{\phi}_{f}}-\frac{1}{\psi\overline{\hat{\phi}}}\left(\frac{\operatorname{var}\left(\theta_{b}\right)-\operatorname{cov}\left(\theta_{b},\theta_{s}\right)-\tilde{\kappa}_{1}}{\left(\left(1+\lambda\right)\mathbb{E}\left[\theta_{s}\right]-\mathbb{E}\left[\theta_{b}\right]\right)}\right)\frac{y}{\overline{\phi}_{l}^{*}},\right.$$
$$\left.\frac{\mathbb{E}\left[\phi_{f}\right]}{\overline{\phi}_{f}}-\frac{1}{\psi\overline{\hat{\phi}}}\left(\frac{\left(1+\lambda\right)\left(\operatorname{var}\left(\theta_{s}\right)-\operatorname{cov}\left(\theta_{b},\theta_{s}\right)\right)-\tilde{\kappa}_{1}}{\left(\left(1+\lambda\right)\mathbb{E}\left[\theta_{s}\right]-\mathbb{E}\left[\theta_{b}\right]\right)}\right)\frac{y}{\overline{\phi}_{l}^{*}}\right\} \tag{26}$$

Recall that

$$\mu_{2}=\frac{\mathbb{E}\left[\phi_{f}\right]}{\overline{\phi}_{f}}-\frac{1}{\psi\overline{\hat{\phi}}}\left[\frac{\operatorname{var}\left(\theta_{s}-\theta_{b}\right)+\lambda\left[\operatorname{var}\left(\theta_{s}\right)-\operatorname{cov}\left(\theta_{s},\theta_{b}\right)\right]-\tilde{\kappa}_{1}}{\left[\left(1+\lambda\right)\mathbb{E}\left[\theta_{s}\right]-\mathbb{E}\left[\theta_{b}\right]\right]}\right]\frac{y}{\overline{\phi}_{l}^{*}}.$$

It is easy to see that $\mu_2^I>\mu_2$. Q.E.D.

Proof of Proposition 8.

Define the threshold

$$\mu_{2}\left(\gamma\right)\equiv\frac{\mathbb{E}\left(\phi_{f}\right)}{\overline{\phi}_{f}}-\frac{1}{\psi}\frac{\left(1-\gamma\right)y}{\overline{\hat{\phi}}_{i}\overline{\phi}_{i}^{*}}\frac{\left(\tilde{\kappa}_{2}-\tilde{\kappa}_{1}\right)}{\left[\left(1+\lambda\right)\mathbb{E}\left[\theta_{s}\right]-\mathbb{E}\left[\theta_{b}\right]\right]},$$

where $\tilde{\kappa}_1$ and $\tilde{\kappa}_2$ were defined in (17) and (19), respectively, and $\overline{\phi}_i^*$ is given by

$$\overline{\phi}_i^* = \frac{\overline{\hat{\phi}_i} + \sqrt{\left(\overline{\hat{\phi}_i}\right)^2 + 4\frac{y}{\psi}(1-\gamma)\left(\overline{\theta}_s - \underline{\theta}_b\right)}}{2}.$$

Note that

$$\mu_2(0) < \mu_2^I < \mu_2(1),$$

where μ_2^I was defined in Proposition 7. Since $\mu_2(\gamma)$ is continuous, there exists $\gamma^* \in (0,1)$ such that $\mu_2(\gamma^*) = \mu_2^I$ (below we show that $\mu_2(\gamma)$ is strictly monotonic, so there is a unique γ^*). Now consider $\gamma < \gamma^*$ and $\mathbb{E}\left(\hat{\phi}_i\right)/\overline{\hat{\phi}}_i \in \left(\mu_2(\gamma), \mu_2^I\right)$. Since $\mathbb{E}\left(\hat{\phi}_i\right)/\overline{\hat{\phi}}_i < \mu_2^I$, an argument identical to that in Proposition 7 establishes that international contracts will always be denominated in foreign currency.

Next, if $\mathbb{E}\left(\hat{\phi}_i\right)/\overline{\hat{\phi}}_i \in \left(\mu_2(\gamma), \mu_2^I\right)$, then there exists an equilibrium in which domestic contracts are denominated in local currency if $\mathcal{H}\left((1-\gamma)y/\overline{\phi}_i^*\right) \geqslant 0$, where $\mathcal{H}(\cdot)$ was defined in the proof of Proposition 2. The reason follows from the proof of Proposition 2, except that in this interval a fraction γ of contracts are international and thus denominated in foreign currency. Simple algebra shows that

$$\mathcal{H}\left((1-\gamma)\frac{y}{\overline{\phi}_i^*}\right) \geqslant 0 \iff \mathbb{E}\left(\hat{\phi}_i\right)/\overline{\hat{\phi}}_i \geqslant \mu_2(\gamma).$$

Thus, we have established the first two parts of the proposition. To see the last part note that

$$\begin{aligned}
\mu_2'(\gamma) &= \frac{1}{\psi}\frac{y}{\overline{\hat{\phi}}_i\overline{\phi}_i^*}\frac{(\tilde{\kappa}_2-\tilde{\kappa}_1)}{[(1+\lambda)\mathbb{E}[\theta_s]-\mathbb{E}[\theta_b]]}\left[1+\frac{(1-\gamma)}{\overline{\phi}_i^*}\frac{\partial\overline{\phi}_i^*}{\partial\gamma}\right]\\
&= \frac{1}{\psi}\frac{y}{\overline{\hat{\phi}}_i\overline{\phi}_i^*}\frac{(\tilde{\kappa}_2-\tilde{\kappa}_1)}{[(1+\lambda)\mathbb{E}[\theta_s]-\mathbb{E}[\theta_b]]}\left[1-\frac{(1-\gamma)}{\overline{\phi}_i^*}\frac{\frac{y}{\psi}\left(\overline{\theta}_s-\underline{\theta}_b\right)}{\sqrt{\left(\overline{\hat{\phi}_i}\right)^2+4\frac{y}{\psi}(1-\gamma)\left(\overline{\theta}_s-\underline{\theta}_b\right)}}\right]\\
&= \frac{1}{\psi}\frac{y}{\overline{\hat{\phi}}_i\overline{\phi}_i^*}\frac{(\tilde{\kappa}_2-\tilde{\kappa}_1)}{[(1+\lambda)\mathbb{E}[\theta_s]-\mathbb{E}[\theta_b]]}\left[\left[\frac{\overline{\hat{\phi}}_i\overline{\phi}_i^*+\frac{y}{\psi}(1-\gamma)\left(\overline{\theta}_s-\underline{\theta}_b\right)}{\overline{\hat{\phi}}_i\overline{\phi}_i^*+2\frac{y}{\psi}(1-\gamma)\left(\overline{\theta}_s-\underline{\theta}_b\right)}\right]\right]
\end{aligned}$$

which is strictly positive since $(\tilde{\kappa}_2-\tilde{\kappa}_1) > 0$ due to Assumption 2. Q.E.D.

Proof of Proposition 9

As before, we can substitute the participation constraint of the seller and the payments feasibility constraint into the objective to write the contracting problem as (ignoring the constant terms)

$$\max_{b_l, b_f} (1+\lambda)\,\mathbb{E}\left[\theta_s \tilde{\phi}_l \left(\frac{\phi_l}{\tilde{\phi}_l} - \frac{\phi_f}{\tilde{\phi}_f}\right) b_l\right] - \mathbb{E}\left[\theta_b \tilde{\phi}_l \left(\frac{\phi_l}{\tilde{\phi}_l} - \frac{\phi_f}{\tilde{\phi}_f}\right) b_l\right],$$

where $\tilde{\phi} = \{\overline{\phi}, \underline{\phi}\}$ depending on whether $b \gtrless \hat{b}$. The first order condition is

$$(1+\lambda)\,\mathbb{E}\left[\theta_s \left(\frac{\phi_l}{\tilde{\phi}_l} - \frac{\phi_f}{\tilde{\phi}_f}\right)\right] - \mathbb{E}\left[\theta_b \left(\frac{\phi_l}{\tilde{\phi}_l} - \frac{\phi_f}{\tilde{\phi}_f}\right)\right] \gtrless 0.$$

First, suppose that $b_l < \hat{b}_l$. Then, after replacing the government's optimal inflation choice, the derivative of the objective function is (recall that $\tilde{\kappa}_2$ is defined in (19)),

$$\left[(1+\lambda)\,\mathbb{E}\left[\theta_s \left(\frac{\hat{\phi} + \frac{1}{\psi}(\theta_s - \theta_b) B_l}{\underline{\hat{\phi}}} - \frac{\phi_f}{\overline{\phi}_f}\right)\right] - \mathbb{E}\left[\theta_b \left(\frac{\hat{\phi} + \frac{1}{\psi}(\theta_s - \theta_b) B_l}{\underline{\hat{\phi}}} - \frac{\phi_f}{\overline{\phi}_f}\right)\right]\right]$$
$$= \left[\left[(1+\lambda)\,\mathbb{E}[\theta_s] - \mathbb{E}[\theta_b]\right]\left(\frac{\mathbb{E}[\hat{\phi}]}{\underline{\hat{\phi}}} - \frac{\mathbb{E}[\phi_f]}{\overline{\phi}_f}\right) + \frac{1}{\psi}\left(\frac{(\tilde{\kappa}_2 + [(1+\lambda)\,\mathbb{E}[\theta_s] - \mathbb{E}[\theta_b]]\,(\mathbb{E}[\theta_s] - \mathbb{E}[\theta_b]))}{\underline{\hat{\phi}}}\right) B_l\right]$$
$$>0$$

so that $b_l < \hat{b}_l$ can never be part of an equilibrium.

Define

$$\kappa_2 \equiv \left[(1+\lambda)\,\mathbb{E}[\theta_s] - \mathbb{E}[\theta_b]\right]\left[\frac{\mathbb{E}[\phi_f]}{\underline{\phi}_f}\left(\overline{\theta}_s - \underline{\theta}_b\right) - \left(\mathbb{E}[\theta_s] - \mathbb{E}[\theta_b]\right)\right]$$
$$+ \left((1+\lambda)\,\mathbb{E}[\theta_s] - \mathbb{E}[\theta_b]\right)\frac{\overline{\hat{\phi}}\,\overline{\phi}_l^*}{y}\left[\frac{\mathbb{E}[\phi_f]}{\underline{\phi}_f} - \frac{\mathbb{E}[\hat{\phi}]}{\overline{\hat{\phi}}}\right] \tag{27}$$

Now, suppose that $b_f < \hat{b}_f$. Then, the derivative of the objective function is

$$\left[(1+\lambda)\mathbb{E}\left[\theta_s\left(\frac{\hat{\phi}+\frac{1}{\psi}(\theta_s-\theta_b)B_l}{\overline{\hat{\phi}}+\frac{1}{\psi}\left(\overline{\theta}_s-\underline{\theta}_b\right)B_l}-\frac{\phi_f}{\underline{\phi}_f}\right)\right]-\mathbb{E}\left[\theta_b\left(\frac{\hat{\phi}+\frac{1}{\psi}(\theta_s-\theta_b)B_l}{\overline{\hat{\phi}}+\frac{1}{\psi}\left(\overline{\theta}_s-\underline{\theta}_b\right)B_l}-\frac{\phi_f}{\underline{\phi}_f}\right)\right]\right]$$

$$=\frac{1}{\psi}\left(\tilde{\kappa}_2-\left[(1+\lambda)\mathbb{E}[\theta_s]-\mathbb{E}[\theta_b]\right]\left[\frac{\mathbb{E}[\phi_f]}{\underline{\phi}_f}\left(\overline{\theta}_s-\underline{\theta}_b\right)-\left(\mathbb{E}[\theta_s]-\mathbb{E}[\theta_b]\right)\right]\right)B_l$$
$$-\left((1+\lambda)\mathbb{E}[\theta_s]-\mathbb{E}[\theta_b]\right)\overline{\hat{\phi}}\left[\frac{\mathbb{E}[\phi_f]}{\underline{\phi}_f}-\frac{\mathbb{E}\left[\hat{\phi}\right]}{\overline{\hat{\phi}}}\right]$$
$$\leqslant\frac{1}{\psi}\left(\tilde{\kappa}_2-\left[(1+\lambda)\mathbb{E}[\theta_s]-\mathbb{E}[\theta_b]\right]\left[\frac{\mathbb{E}[\phi_f]}{\underline{\phi}_f}\left(\overline{\theta}_s-\underline{\theta}_b\right)-\left(\mathbb{E}[\theta_s]-\mathbb{E}[\theta_b]\right)\right]\right)\frac{y}{\overline{\phi}_l^*}$$
$$-\left((1+\lambda)\mathbb{E}[\theta_s]-\mathbb{E}[\theta_b]\right)\overline{\hat{\phi}}\left[\frac{\mathbb{E}[\phi_f]}{\underline{\phi}_f}-\frac{\mathbb{E}\left[\hat{\phi}\right]}{\overline{\hat{\phi}}}\right]$$
$$<0$$

where the last inequality follows from Assumption 3. Q.E.D.

C Additional Results and Extensions

C.1 TNT Model with Endogenous Real Exchange Rate Risk

This section shows that exogenous risk in the price of foreign currency $\mathbb{E}[\phi_f]/\overline{\phi}_f$ can arise in an extension of our model with tradable and non-tradable goods and shocks to the relative demand of these goods in the domestic economy. Suppose the numeraire good in our model is a composite of tradable and non-tradable goods, $c = c_T^{\alpha} c_N^{1-\alpha}$, where c_T (respectively, c_N) is the domestic consumption of tradables (respectively, non-tradables), and α is a stochastic parameter that captures shocks to the relative demand of these goods. The equivalent good in the foreign country is given by $c^* = \left(c_T^*\right)^{\alpha^*}\left(c_N^*\right)^{1-\alpha^*}$. We assume that α^* is deterministic. We also normalize the endowments $y_T = y_N = y_T^* = y_N^* = y$. Consistent with our baseline model, we denote the price of the local (respectively, foreign) currency in terms of the domestic composite by ϕ_l (respectively, ϕ_f). Additionally, we normalize the price of the foreign currency in terms of the foreign composite good to 1. The exchange rate e is defined as the price of the foreign currency in terms of the local currency. Let p_T denote the price of the tradable goods in the domestic economy in terms of the local currency and p_T^* denote the price of the tradable goods in the foreign economy in terms of the foreign currency.

Given the Cobb-Douglas structure, p_T and p_T^* are given by

$$p_T = \frac{1}{\phi_l}\alpha\left(\frac{c_N}{c_T}\right)^{1-\alpha} \text{ and } p_T^* = \alpha^*\left(\frac{c_N^*}{c_T^*}\right)^{1-\alpha^*}.$$

In this model, the law of one price for tradable goods holds. Market clearing in all goods implies that the exchange rate e is given by

$$e = \frac{p_T}{p_T^*} = \frac{\alpha}{\alpha^*}\frac{1}{\phi_l}.$$

Therefore,

$$\phi_f = e\phi_l = \frac{\alpha}{\alpha^*}.$$

Thus, we can generate fluctuations in the real exchange rate (the price of the foreign currency in terms of the domestic composite good, ϕ_f) by assuming a stochastic process for α.

C.2 A Model with Centralized Markets

In this section, we show that our formulation with bilateral contracts is identical to a model with centralized debt markets in both currencies. Consider an economy with buyers, sellers, and a Walrasian market for debt in local and foreign currencies. For this setting it is natural to assume that buyers will be "debtors" and sellers will be "creditors". There are two periods $t = 1, 2$. The preferences of buyers are given by $U_b = (1+\lambda)\, c_{1b} + \mathbb{E}\left[\theta_b c_{2b}\right]$, and the preferences of sellers are given by $U_s = c_{1s} + \mathbb{E}\left[\theta_s c_{2s}\right]$, where c_{ti} denotes the consumption of the numeraire good by agent $i = b, s$ in periods $t = 1, 2$. Agents can borrow and lend in either currency at nominal interest rates R_c, $c = l, f$. That is, an agent can exchange $1/R_c$ units of currency c in period 1, for a unit of currency c in period 2. Without loss of generality, we normalize the period-1 prices of local and foreign currency in terms of the numeraire good to 1, and denote the prices in period 2 by ϕ_c, $c = l, f$. The agents' budget constraints are given by

$$c_{1i} = y + \frac{b_{li}}{R_l} + \frac{b_{fi}}{R_f},$$

in period one and

$$c_{2i} = y - \phi_l b_{lc} - \phi_f b_{fc}$$

in period 2, where y is the endowment of goods, which we set to be equal across agents and time, and b_{ci} is the debt denominated in currency c issued in period 1 by agent i. The buyer's problem is to choose debt in local and foreign currencies, $b_{lb} \geqslant 0$ and $b_{fb} \geqslant 0$, to maximize U_b subject to the budget constraint and the payments feasibility constraint $y - \phi_l b_l - \phi_f b_f \geqslant 0$, for all price realizations. In this context, one can interpret the payments feasibility constraint as a borrowing constraint. The seller's problem is to choose savings in local and foreign currencies, $b_{lb} \leqslant 0$ and $b_{fb} \leqslant 0$, to maximize U_s subject to the budget constraint. We assume that debt issued by the buyers is non-negative in either currency as in the baseline model. Note that Assumption 1 guarantees that $c_{1b} > y$ and $c_{1s} < y$ in equilibrium. Finally, the government's problem in this economy is identical to that in the baseline economy.

The following proposition states that the equilibrium allocation in this economy coincides with that in the baseline economy with bilateral contracts.

Proposition 10. *The equilibrium allocation in the economy with centralized markets is identical to that in the baseline economy with bilateral contracts, i.e.,* $B_{lb} = -B_{ls} = B_l$ *and* $B_{fb} = -B_{fs} = B_f$. *In addition, the equilibrium interest rates in the economy with centralized markets are given by* $R_c = (\mathbb{E}[\theta_s \phi_c])^{-1}$, *for* $c \in \{l, f\}$.

Proof. Conjecture that equilibrium interest rates are given by $R_c = (\mathbb{E}[\theta_s \phi_c])^{-1}$. Then, Assumption 1 implies that $(1+\lambda)/R_f \geqslant \mathbb{E}[\theta_b \phi_f]$, which, in turn, implies that the discount factor of the borrower is greater than or equal to the real interest rate in foreign currency. Therefore, the payments feasibility constraint is binding and we have that $b_f = y/\overline{\phi}_f - \overline{\phi}_l b_l / \overline{\phi}_f$. Thus, the problem for the buyer is

$$\max_{b_f \geqslant 0, b_l \geqslant 0} (1+\lambda)\left(y_0 + \frac{b_{lb}}{R_l} + \frac{1}{R_f}\left[\frac{y}{\overline{\phi}_f} - \frac{\overline{\phi}_l b_{lb}}{\overline{\phi}_f}\right]\right) + \mathbb{E}\left[\theta_b \left(y - \phi_l b_{lb} - \phi_f \left[\frac{y}{\overline{\phi}_f} - \frac{\overline{\phi}_l b_{lb}}{\overline{\phi}_f}\right]\right)\right].$$

The solution to this problem is

$$\begin{cases} b_{lb} = \frac{y}{\overline{\phi}_l}, b_{fb} = 0, & \text{if } (1+\lambda)\frac{1}{R_l \overline{\phi}_l} - \mathbb{E}\left[\theta_b \frac{\phi_l}{\overline{\phi}_l}\right] > (1+\lambda)\frac{1}{R_f \overline{\phi}_f} - \mathbb{E}\left[\theta_b \frac{\phi_f}{\overline{\phi}_f}\right] \\ b_{lb} = 0, b_{fb} = \frac{y}{\overline{\phi}_f}, & \text{otherwise.} \end{cases} \tag{28}$$

The seller's problem is

$$\max_{b_f \geqslant 0, b_l \geqslant 0} \left(y + \frac{b_{ls}}{R_l} + \frac{b_{fs}}{R_f}\right) + \mathbb{E}\left[\theta_s \left(y - \phi_l b_{ls} - \phi_f b_{fs}\right)\right].$$

This problem is linear, and the seller will choose infinite savings in currency c if $1/R_c < \mathbb{E}[\theta_s \phi_c]$, any savings if $1/R_c = \mathbb{E}[\theta_s \phi_c]$ and zero otherwise. This implies that for markets to clear we must have that $1/R_c = \mathbb{E}[\theta_s \phi_c]$, for both currencies c. If we substitute the equilibrium interest rates into (28), we get a condition identical to that in Proposition 1. Consequently, the equilibrium allocation in the two environments are identical. Q.E.D.

Analyzing the economy with centralized debt markets allows us to characterize the currency choice of debt, the equilibrium levels of debt and interest rates, through the lens of supply and demand. Equilibrium quantities of debt are determined by the borrowing constraint of borrowers and the equilibrium interest rates are determined by the sellers' indifference condition. That is, the interest rate in a given currency is determined by the seller's taste shock and how it covaries with inflation. Given that the inflation choice of the government is endogenous, this implies that the use of local currency debt and the level of policy risk affects the equilibrium interest rate in local currency. For related empirical evidence, see Kalemli-Ozcan et al. (2019), who argue that policy risk is a relevant determinant of interest rates.

C.3 Microfoundation of Inflation Loss Function

Consider an extension of the baseline model in which, in addition to buyers and sellers, there are households. Households derive utility from the consumption of the numeraire good c_h, a cash good z, and disutility from exerting labor n in the second period captured by the utility function

$$U_h = \omega z + c_h - \frac{n^2}{2},$$

where $\omega > 1$. Households are endowed with money claims on the government m and need to pay ad-valorem taxes τ on labor income. The households' budget constraint is given by

$$c_h + p_z z = w(1-\tau)n + m\phi_l, \tag{29}$$

where p_z is the price of the cash good, w is the real wage, and ϕ_l is the price of money, all expressed in terms of the numeraire good. Households also face a cash-in-advance constraint which requires that the cash good needs to be purchased with money holdings

$$p_z z \leqslant m\phi_l. \tag{30}$$

The government uses taxes to finance government expenditure g (expressed in terms of the numeraire good) and repay money claims. Government expenditures are unknown in the initial period and drawn from a distribution with bounded support $[\underline{g}, \overline{g}]$. The government budget constraint is given by

$$g + m\phi_l = w\tau n. \tag{31}$$

Finally, we assume that both the numeraire good and the cash good can be produced with a linear technology that uses labor $n = c_h + g + z$. Free entry of firms implies that $w = p_z = 1$. The problem of the household is to maximize U_h subject to (29) and (30). We conjecture (and verify later) that the cash-in-advance constraint binds and thus the solution to the household problem is $n = (1-\tau)$, $c_h = (1-\tau)^2$, and $z = m\phi_l$. Define the target tax rate $\hat{\tau}$ and level of inflation $\hat{\phi}$ as the tax rate and the level of inflation that maximize the household's utility, subject to the allocations defined above and (31). The target tax rate is given by $\hat{\tau} = (\omega - 1)/(2\omega - 1)$. The target level of inflation is given by

$$\hat{\phi} = \frac{\hat{\tau}(1-\hat{\tau}) - g}{m}. \tag{32}$$

Note that the target tax rate is independent of g, which implies that shocks to government spending are absorbed with seigniorage. In order to guarantee that $\hat{\phi} \geqslant 0$, we assume that $\overline{g} < \hat{\tau}(1-\hat{\tau})$. Finally, notice that since $\hat{\tau} \leqslant 1$ and households strictly prefer the cash good to the numeraire good ($\omega > 1$), the cash-in-advance constraint always binds.

If we compute a second order approximation to the household's utility around the target government policies, we obtain

$$U_h = const - \frac{(2\omega-1)\,2m^2}{(1-2\hat{\tau})^2}\left(\phi_l - \hat{\phi}\right)^2,$$

where $const \equiv \omega\left(\hat{\tau}(1-\hat{\tau}) - g\right) + (1-\hat{\tau})^2/2$. It follows that the loss function in the baseline model maps into a second order approximation of the household's utility. In particular, the inflation cost parameter is given by $\psi \equiv 2\,(2\omega-1)\,2m^2/\,(1-2\hat{\tau})^2 > 0$ and $\hat{\phi}$ is given by (32), which implies that shocks to the inflation target can be microfounded by shocks to government expenditure. This interpretation of policy risk, together with the results on the characterization of the set of competitive equilibria, can shed light on why countries with more volatile government expenditures tend to have more domestic dollarization, as shown in Figure D.1.

C.4 Relaxing Assumption 2

In this section, we characterize the equilibrium outcomes and the solution to the social planner's problem when the converse of Assumption 2 holds. For simplicity, we assume that the distributions for θ_s and θ_b are i.i.d. An almost identical argument holds for the general case. Define the following two constants

$$\nu_1 \equiv (1+\lambda)\,\text{var}\,(\theta) - \lambda \mathbb{E}\,[\theta]\left(\frac{\mathbb{E}\,[\phi_f]}{\overline{\phi}_f}\left(\overline{\theta} - \underline{\theta}\right)\right)$$

and

$$\nu_2 \equiv (2+\lambda)\,\text{var}(\theta) - \lambda \mathbb{E}[\theta]\left(\frac{\mathbb{E}[\phi_f]}{\overline{\phi}_f}\left(\overline{\theta}-\underline{\theta}\right)\right).$$

Note that Assumption 2 for the i.i.d. case is $\nu_1 > 0$. The next proposition characterizes the set of equilibrium outcomes when $\nu_1 \leqslant 0$.

Proposition 11. *Suppose that Assumption 1 holds and $\nu_1 \leqslant 0$. Then, if $\nu_2 > 0$ the equilibrium outcomes are identical to Proposition 2. If $\nu_2 \leqslant 0$, then there exists thresholds $\mu_1 = \mathbb{E}[\phi_f]/\overline{\phi}_f$ and $\tilde{\mu}_2 > \mu_1$ such that*

1. *If $\mathbb{E}\left[\hat{\phi}\right]/\overline{\hat{\phi}} \geqslant \tilde{\mu}_2$, there exists a unique equilibrium in which $B_l = y/\overline{\phi}_l^*$ where $\overline{\phi}_l^*$ is the positive solution to*

$$\overline{\phi}_l^* = \overline{\hat{\phi}} + \frac{1}{\psi}\left(\overline{\theta}-\underline{\theta}\right)\frac{y}{\overline{\phi}_l^*}.$$

2. *If $\mu_1 < \mathbb{E}\left[\hat{\phi}\right]/\overline{\hat{\phi}} < \tilde{\mu}_2$, there exists a unique interior equilibrium with $B_l \in \left(0, y/\overline{\phi}_l^*\right)$.*

3. *If $\mathbb{E}\left[\hat{\phi}\right]/\overline{\hat{\phi}} \leqslant \mu_1$, there exists a unique equilibrium in which $B_l = 0$.*

Proof of Proposition 11

As in the baseline environment, the participation constraint for the seller will bind and that of the buyer will be slack. Thus, as in the proof of Proposition 2, the following definitions will be useful. Define

$$\mathcal{H}(B) \equiv (1+\lambda)\, M_2(B) - M_1(B),$$

where

$$\begin{aligned} M_2(B) &\equiv \mathbb{E}\left[\theta_s\left(\phi_l(B) - \frac{\phi_f}{\overline{\phi}_f}\overline{\phi}_l(B)\right)\right] \\ &= \mathbb{E}(\theta_s)\,\overline{\hat{\phi}}\left(\frac{\mathbb{E}\left(\hat{\phi}\right)}{\overline{\hat{\phi}}} - \frac{\mathbb{E}(\phi_f)}{\overline{\phi}_f}\right) \\ &+ \frac{1}{\psi}\left(\text{var}(\theta) - \frac{\mathbb{E}(\phi_f)}{\overline{\phi}_f}\mathbb{E}(\theta)\left(\overline{\theta}-\underline{\theta}\right)\right) B_l \end{aligned}$$

and

$$
\begin{aligned}
M_1(B) &\equiv \mathbb{E}\left[\theta_b\left(\phi_l(B) - \frac{\phi_f}{\overline{\phi}_f}\overline{\phi}_l(B)\right)\right] \\
&= \mathbb{E}(\theta_b)\,\overline{\hat{\phi}}\left(\frac{\mathbb{E}(\hat{\phi})}{\overline{\hat{\phi}}} - \frac{\mathbb{E}(\phi_f)}{\overline{\phi}_f}\right) \\
&\quad - \frac{1}{\psi}\left(\mathrm{var}(\theta) + \frac{\mathbb{E}(\phi_f)}{\overline{\phi}_f}\mathbb{E}(\theta)\left(\overline{\theta} - \underline{\theta}\right)\right) B_l,
\end{aligned}
$$

where we have used the best response of the government

$$
\phi_l(B) = \hat{\phi} + \frac{1}{\psi}(\theta_b - \theta_s) B_l.
$$

It will also be useful to compute

$$
M_1'(B) = -\frac{1}{\psi}\left[\mathrm{var}(\theta) + \frac{\mathbb{E}(\phi_f)}{\overline{\phi}_f}\mathbb{E}(\theta)\left(\overline{\theta} - \underline{\theta}\right)\right]
$$

and

$$
M_2'(B) = \frac{1}{\psi}\left(\mathrm{var}(\theta) - \frac{\mathbb{E}(\phi_f)}{\overline{\phi}_f}\mathbb{E}(\theta)\left(\overline{\theta} - \underline{\theta}\right)\right).
$$

There are three types of equilibria that can exist. First, an equilibrium with $B_l = 0$ (full foreign currency equilibrium) exists if and only if $\mathcal{H}(0) \leqslant 0$. Next, an equilibrium in which $B_f = 0$ (full local currency equilibrium) can exist if and only if $\mathcal{H}\left(y/\overline{\phi}_l^*\right) \geqslant 0$, where $y/\overline{\phi}_l^*$ corresponds to the maximal feasible value of B_l, and $\overline{\phi}_l^*$ solves

$$
\overline{\phi}_l^* = \overline{\hat{\phi}} + \frac{1}{\psi}\left(\overline{\theta} - \underline{\theta}\right)\frac{y}{\overline{\phi}_l^*}
$$

or

$$
\overline{\phi}_l^* = \frac{\overline{\hat{\phi}} + \sqrt{\left(\overline{\hat{\phi}}\right)^2 + 4\frac{y}{\psi}\left(\overline{\theta} - \underline{\theta}\right)}}{2}.
$$

Finally, an interior equilibrium exists if and only if there exists some $B_l \in \left(0, y/\overline{\phi}_l^*\right)$ such that $\mathcal{H}(B_l) = 0$.

First suppose that $\nu_2 > 0$. The characterization of equilibria in this case follows the proof of Proposition 2, by noting that

$$
\mathcal{H}'(B) = (1+\lambda) M_2'(B) - M_1'(B) = \frac{1}{\psi}\left[(2+\lambda)\,\mathrm{var}(\theta) - \lambda\frac{\mathbb{E}(\phi_f)}{\overline{\phi}_f}\mathbb{E}(\theta)\left(\overline{\theta} - \underline{\theta}\right)\right] > 0
$$

since $\nu_2 > 0$.

Now suppose that $\nu_2 < 0$. The case with $\nu_2 = 0$ is trivial and will be proved at the end. Define $\mu_1 \equiv \mathbb{E}[\phi_f] / \overline{\phi}_f$ and

$$\tilde{\mu}_2 \equiv \frac{\mathbb{E}[\phi_f]}{\overline{\phi}_f} - \frac{1}{\psi} \frac{\nu_2}{\lambda \mathbb{E}(\theta) \overline{\hat{\phi}}} \frac{y}{\overline{\phi}_l^*}.$$

Note that $\tilde{\mu}_2 > \mu_1$ since $\nu_2 < 0$. Suppose that $\mathbb{E}\left[\hat{\phi}\right] / \overline{\hat{\phi}} \geqslant \tilde{\mu}_2$. Then,

$$\mathcal{H}\left(\frac{y}{\overline{\phi}_l^*}\right) = \lambda \mathbb{E}(\theta) \overline{\hat{\phi}} \left(\frac{\mathbb{E}\left[\hat{\phi}\right]}{\overline{\hat{\phi}}} - \frac{\mathbb{E}[\phi_f]}{\overline{\phi}_f}\right) + \frac{1}{\psi}\left[(2+\lambda)\operatorname{var}(\theta) - \lambda \frac{\mathbb{E}(\phi_f)}{\overline{\phi}_f} \mathbb{E}(\theta)\left(\overline{\theta} - \underline{\theta}\right)\right] \frac{y}{\overline{\phi}_l^*} \geqslant 0.$$

Therefore,

$$\mathcal{H}(B) > \mathcal{H}\left(\frac{y}{\overline{\phi}_l^*}\right) \geqslant 0,$$

since $\mathcal{H}'(B) < 0$ for all $B \leqslant y/\overline{\phi}_l^*$ due to the fact that $\nu_2 < 0$. Thus, there exists a unique full local currency equilibrium. Now suppose that $\mu_1 < \mathbb{E}\left[\hat{\phi}\right] / \overline{\hat{\phi}} < \tilde{\mu}_2$. Then, we have $\mathcal{H}(0) > 0$, $\mathcal{H}\left(y/\overline{\phi}_l^*\right) < 0$, and $\mathcal{H}'(B) < 0$. Therefore, since $\mathcal{H}$ is continuous, we have a unique interior equilibrium. Finally, suppose that $\mathbb{E}\left[\hat{\phi}\right] / \overline{\hat{\phi}} < \mu_1$. Then, we have $\mathcal{H}(B) < \mathcal{H}(0) \leqslant 0$. Therefore, there exists a unique full foreign currency equilibrium.

Finally, suppose that $\nu_2 = 0$. Then, $\mu_1 = \tilde{\mu}_2$ and so there exists a unique full local equilibrium if $\mathbb{E}\left[\hat{\phi}\right] / \overline{\phi} > \mu_1$ and a unique full foreign currency equilibrium otherwise. Q.E.D.

This proposition shows that there are two possibilities if Assumption 2 is violated; in the first, the set of equilibrium outcomes is identical to Proposition 2, while in the second, for an intermediate range of policy risk there is a unique interior equilibrium. In summary, relaxing Assumption 2 does not significantly affect the set of competitive equilibria.

We now turn to the characterization of the planner's problem.

Proposition 12. *Suppose that Assumption 1 holds and $\nu_1 \leqslant 0$. Then, there exists a threshold $\tilde{\mu}_{SP}$, with $\tilde{\mu}_2 < \tilde{\mu}_{SP} \leqslant \mu_1$, such that:*

1. *If $\mathbb{E}\left[\hat{\phi}\right] / \overline{\hat{\phi}} \geqslant \tilde{\mu}_{SP}$, then the solution to the social planner's problem is $B_l^{sp} = y/\overline{\phi}_l^*$, where $\overline{\phi}_l^*$ was defined in Proposition 2.*
2. *If $\mu_1 < \mathbb{E}\left[\hat{\phi}\right] / \overline{\hat{\phi}} < \tilde{\mu}_{SP}$, then there exists a unique interior equilibrium with $B_l^{sp} \in \left(0, y/\overline{\phi}_l^*\right)$.*
3. *If $\mathbb{E}\left[\hat{\phi}\right] / \overline{\hat{\phi}} \leqslant \mu_1$, then the solution to the social planner's problem is $B_l^{sp} = 0$.*

Proof of Proposition 12.

As in the baseline model, the participation constraint for the seller will bind and that for the buyer will be slack. Thus, we can write the planner's problem as

$$\max_{B_l}\left(\mathbb{E}\left([(1+\lambda)\theta_s-\theta_b]\left(\left(\phi_l-\frac{\phi_f}{\overline{\phi}_f}\overline{\phi}_l\right)B_l+\frac{\phi_f}{\overline{\phi}_f}y\right)\right)+2y\right)-l(\phi_l),$$

subject to the optimal government's policy in (8) and (9). Given our previous definitions, it will be useful to define the planning problem as follows:

$$SP(B)\equiv\max_{B}\left[(1+\lambda)M_2(B)B-M_1(B)B-\mathbb{E}l(\phi_l(B))\right]+\tilde{y},$$

where $\tilde{y}\equiv\mathbb{E}[\theta]\left(2+\lambda\mathbb{E}[\phi_f]/\overline{\phi}_f\right)y$, subject to

$$\phi_l(B)=\hat{\phi}+\frac{1}{\psi}(\theta_s-\theta_b)B.$$

The first derivative of the objective function is

$$SP'(B)=\left[(1+\lambda)M_2(B)-M_1(B)+\Delta(B)B\right],$$

where we have used the definition of $l(\phi)$ and

$$\begin{aligned}\Delta(B)&\equiv(1+\lambda)M_2'(B)-M_1'(B)-\mathbb{E}(\theta_s-\theta_b)\phi_l'(B)\\&=\frac{1}{\psi}\lambda\left(\mathrm{var}(\theta)-\frac{\mathbb{E}[\phi_f]}{\overline{\phi}_f}\mathbb{E}(\theta)\left(\overline{\theta}-\underline{\theta}\right)\right).\end{aligned}$$

Next, let us check the second derivative of the planner's problem. First, we have

$$\Delta'(B)=(1+\lambda)M_2''(B)-M_1''(B)-\mathbb{E}(\theta_s-\theta_b)\phi_l''(B)=0,$$

which implies that

$$\begin{aligned}SP''(B)&=(1+\lambda)M_2'(B)-M_1'(B)+\Delta(B)\\&=\frac{2}{\psi}\left((1+\lambda)\mathrm{var}(\theta)-\lambda\frac{\mathbb{E}[\phi_f]}{\overline{\phi}_f}\mathbb{E}(\theta)\left(\overline{\theta}-\underline{\theta}\right)\right),\end{aligned}$$

which is negative since $\nu_1\leqslant 0$. Thus, the planner's problem is concave and the optimal solution, if interior, is given by the first order condition

$$(1+\lambda)M_2(B)-M_1(B)+\Delta(B)B=0.$$

Let us consider the FOC

$$\lambda\mathbb{E}[\theta]\,\overline{\hat{\phi}}\left(\frac{\mathbb{E}\left[\hat{\phi}\right]}{\overline{\hat{\phi}}}-\frac{\mathbb{E}[\phi_f]}{\overline{\phi}_f}\right)+\frac{1}{\psi}\left[(2+\lambda)\operatorname{var}(\theta)-\lambda\frac{\mathbb{E}[\phi_f]}{\overline{\phi}_f}\mathbb{E}(\theta)\left(\overline{\theta}-\underline{\theta}\right)\right]B$$
$$+\frac{1}{\psi}\lambda\left(\operatorname{var}(\theta)-\frac{\mathbb{E}[\phi_f]}{\overline{\phi}_f}\mathbb{E}(\theta)\left(\overline{\theta}-\underline{\theta}\right)\right)B$$
$$=\lambda\mathbb{E}[\theta]\,\overline{\hat{\phi}}\left(\frac{\mathbb{E}\left[\hat{\phi}\right]}{\overline{\hat{\phi}}}-\frac{\mathbb{E}[\phi_f]}{\overline{\phi}_f}\right)+\frac{1}{\psi}2\left[(1+\lambda)\operatorname{var}(\theta)-\lambda\frac{\mathbb{E}[\phi_f]}{\overline{\phi}_f}\mathbb{E}(\theta)\left(\overline{\theta}-\underline{\theta}\right)\right]B.$$

Then, if

$$\frac{\mathbb{E}\left[\hat{\phi}\right]}{\overline{\hat{\phi}}}\geqslant\frac{\mathbb{E}[\phi_f]}{\overline{\phi}_f}-\frac{2}{\psi\lambda\mathbb{E}[\theta]\,\overline{\hat{\phi}}}\left[(1+\lambda)\operatorname{var}(\theta)-\lambda\frac{\mathbb{E}[\phi_f]}{\overline{\phi}_f}\mathbb{E}(\theta)\left(\overline{\theta}-\underline{\theta}\right)\right]\frac{y}{\overline{\phi}_l^*}$$

we have

$$B_l=\frac{y}{\overline{\phi}_l^*}.$$

If instead

$$\frac{\mathbb{E}[\phi_f]}{\overline{\phi}_f}<\frac{\mathbb{E}\left[\hat{\phi}\right]}{\overline{\hat{\phi}}}<\frac{\mathbb{E}[\phi_f]}{\overline{\phi}_f}-\frac{2}{\psi\lambda\mathbb{E}[\theta]\,\overline{\hat{\phi}}}\left[(1+\lambda)\operatorname{var}(\theta)-\lambda\frac{\mathbb{E}[\phi_f]}{\overline{\phi}_f}\mathbb{E}(\theta)\left(\overline{\theta}-\underline{\theta}\right)\right]\frac{y}{\overline{\phi}_l^*},$$

then B_l^{sp} is interior and given by

$$B^{sp}=\frac{-\lambda\mathbb{E}[\theta]\,\overline{\hat{\phi}}\left(\frac{\mathbb{E}[\hat{\phi}]}{\overline{\hat{\phi}}}-\frac{\mathbb{E}[\phi_f]}{\overline{\phi}_f}\right)}{\frac{1}{\psi}2\left[(1+\lambda)\operatorname{var}(\theta)-\lambda\frac{\mathbb{E}[\phi_f]}{\overline{\phi}_f}\mathbb{E}(\theta)\left(\overline{\theta}-\underline{\theta}\right)\right]}.$$

If

$$\frac{\mathbb{E}\left[\hat{\phi}\right]}{\overline{\hat{\phi}}}\leqslant\frac{\mathbb{E}[\phi_f]}{\overline{\phi}_f},$$

then $B_l^{sp}=0$. Q.E.D.

Finally, we provide a comparison between the competitive equilibria and the social planner's solution. The main result here is that if Assumption 2 is violated then the planner always prefers weakly less local currency than the competitive equilibrium.

Proposition 13. *Suppose that* $\nu_1\leqslant 0$. *Then* $B_l^{sp}\leqslant B_l^{ce}$.

Proof of Proposition 13

Let us first consider the case in which $\nu_1<0$. The case with $\nu_1=0$ is trivial and will be considered at the end. Suppose for contradiction that $B_l^{sp}>B_l^{ce}$. There are two cases to consider; $B_l^{ce} = 0$ or B_l^{ce} interior. Suppose first that $B_l^{ce} = 0$. Then it must be that $\mathcal{H}(0) \leqslant 0$ and so $\mathbb{E}\left[\hat{\phi}\right]/\overline{\hat{\phi}} \leqslant \mathbb{E}\left[\phi_f\right]/\overline{\phi}_f$. Next, if $B_l^{sp} > 0$, from the proof of Proposition 12 it must be that

$$\mathcal{S}\left(B_l^{sp}\right) \equiv \lambda \mathbb{E}\left[\theta\right] \overline{\hat{\phi}} \left(\frac{\mathbb{E}\left[\hat{\phi}\right]}{\overline{\hat{\phi}}} - \frac{\mathbb{E}\left[\phi_f\right]}{\overline{\phi}_f} \right) + \frac{1}{\psi} 2 \left[(1+\lambda) \operatorname{var}(\theta) - \lambda \frac{\mathbb{E}\left(\phi_f\right)}{\overline{\phi}_f} \mathbb{E}\left(\theta\right) \left(\overline{\theta} - \underline{\theta}\right) \right] B_l^{sp} \geqslant 0.$$

However, since $\nu_1 < 0$ and $\mathbb{E}\left[\hat{\phi}\right]/\overline{\hat{\phi}} \leqslant \mathbb{E}\left[\phi_f\right]/\overline{\phi}_f$, $\mathcal{S}\left(B_l^{sp}\right) < 0$ yielding a contradiction. Second, suppose $B_l^{ce} > 0$ and interior. Then, it must be that $\mathcal{H}\left(B_l^{ce}\right) = 0$. We need to consider two more subcases here; $\nu_2 \geqslant 0$ and $\nu_2 < 0$. Suppose that $\nu_2 > 0$. Then, since

$$\mathcal{H}\left(B_l^{ce}\right) = \lambda \mathbb{E}\left[\theta\right] \overline{\hat{\phi}} \left(\frac{\mathbb{E}\left[\hat{\phi}\right]}{\overline{\hat{\phi}}} - \frac{\mathbb{E}\left[\phi_f\right]}{\overline{\phi}_f} \right) + \frac{1}{\psi} \left[(2+\lambda) \operatorname{var}(\theta) - \lambda \frac{\mathbb{E}\left(\phi_f\right)}{\overline{\phi}_f} \mathbb{E}\left(\theta\right) \left(\overline{\theta} - \underline{\theta}\right) \right] B_l^{ce} = 0$$

it must be that $\mathbb{E}\left[\hat{\phi}\right]/\overline{\hat{\phi}} \leqslant \mathbb{E}\left[\phi_f\right]/\overline{\phi}_f$. Therefore, $\mathcal{S}\left(B_l^{sp}\right) < 0$ yielding a contradiction. Next, suppose that $\nu_2 \leqslant 0$. Then,

$$\mathcal{S}\left(B_l^{sp}\right) < \mathcal{H}\left(B_l^{sp}\right) \leqslant \mathcal{H}\left(B_l^{ce}\right) = 0,$$

where the first inequality follows from $\nu_1 < 0$ and second from $\nu_2 \leqslant 0$ and $B_l^{sp} > B_l^{ce}$. This is a contradiction.

Finally consider the case when $\nu_1 = 0$. From the proofs of Propositions 11 and 12 it is easy to see that $B_l^{ce} = B_l^{sp} = y/\overline{\hat{\phi}}^*$ if $\mathbb{E}\left[\hat{\phi}\right]/\overline{\hat{\phi}} > \mathbb{E}\left[\phi_f\right]/\overline{\phi}_f$ and $B_l^{ce} = B_l^{sp} = 0$ otherwise. Q.E.D.

Depending on the level of policy risk, the inequality $B_l^{sp} \leqslant B_l^{ce}$ can be strict or hold with an equality. For example, if $\mathbb{E}\left[\hat{\phi}\right]/\overline{\hat{\phi}} \geqslant \tilde{\mu}_{SP}$ then $B_l^{sp} = B_l^{ce} = y/\overline{\phi}_l^*$. On the other hand, if $\nu_2 \leqslant 0$ and

$$\tilde{\mu}_2 \leqslant \frac{\mathbb{E}\left[\hat{\phi}\right]}{\overline{\hat{\phi}}} < \tilde{\mu}_{sp},$$

then $B_l^{sp} < B_l^{ce}$.

Recall that Assumption 2 guarantees that the covariance benefits of denominating in local currency dominates the additional increase in price risk. If Assumption 2 is violated, then these covariance benefits are relatively small which implies that the planner always prefers weakly less local currency than the private agents. As in the baseline model, this arises because private agents do not internalize the effects of their choices on the inflation losses.

C.5 Model with Commitment

In this section, we describe a model in which the government can commit. The main result is that in this case the competitive equilibrium is constrained efficient. Consequently, there is no role for policy in regulating private contracts. To see this, let $\sigma = \left(\hat{\phi}, \phi_f, \theta_s, \theta_b\right)$ denote the state of the world after shocks are realized in period 2. We consider a government who first chooses a state contingent policy $\phi_l(\sigma)$ and then private agents make their decisions. Finally, the state of the world is realized and policy $\phi_l(\sigma)$ is implemented.

Define $P(\phi)$ as

$$P(\phi) \equiv \max_{c_s(\sigma), c_b(\sigma), x, b_l \geqslant 0, b_f \geqslant 0} (1+\lambda)\, x + \mathbb{E}\left[\theta_b c_b(\sigma)\right] - x + \mathbb{E}\left[\theta_s c_s(\sigma)\right]$$

subject to

$$c_s(\sigma) = y + \phi_f b_f + \phi_l(\sigma)\, b_l,$$

$$c_b(\sigma) = y - \phi_f b_f - \phi_l(\sigma)\, b_l,$$

the participation constraints of the buyer and seller, and

$$y - \phi_l(\sigma)\, b_l - \phi_f b_f \geqslant 0, \ \forall \sigma.$$

This is the problem that private agents solve and this characterizes the optimal private contract. Then, the problem with commitment can be written as

$$\max_{\phi(\sigma)} P(\phi) - \frac{1}{2}\psi \mathbb{E}\left(\phi_l(\sigma) - \hat{\phi}\right)^2.$$

Next, recall the planning problem:

$$\max_{\phi(\sigma), C_s(\sigma)\geqslant 0, C_b(\sigma)\geqslant 0, x, B_l \geqslant 0, B_f \geqslant 0} (1+\lambda)\, x + \mathbb{E}\left[\theta_b C_b(\sigma)\right] - x + \mathbb{E}\left[\theta_s C_s(\sigma)\right] - \frac{1}{2}\psi \mathbb{E}\left(\phi_l(\sigma) - \hat{\phi}\right)^2$$

subject to

$$C_s(\sigma) = y + \phi_f B_f + \phi_l(\sigma)\, B_l,$$

$$C_b(\sigma) = y - \phi_f B_f - \phi_l(\sigma)\, B_l,$$

the participation constraints of buyers and sellers, and

$$y - \phi_l(\sigma)\, B_l - \phi_f B_f \geqslant 0, \ \forall \sigma.$$

Thus, when $c_j(\sigma) = C_j(\sigma)$, $j \in \{b, s\}$ and $b_k = B_k, k \in \{l, f\}$, this problem can be written as

$$\max_{\phi_l(\sigma)} P(\phi) - \frac{1}{2}\psi\mathbb{E}\left(\phi_l(\sigma) - \hat{\phi}\right)^2,$$

which is identical to the problem with commitment. Thus, the proof of the following proposition is immediate.

Proposition 14. *Suppose the government can commit to a state contingent policy $\phi_l(\sigma)$. Then the competitive equilibrium is constrained efficient.*

C.6 Model with International Trade Contracts: Generalized Result

This section shows that the result in Proposition 7 can be generalized to the case when international trade contracts can be set in different currencies, under a parametric assumption. For simplicity, we assume that θ_s and θ_b are independent and identically distributed with $\text{var}(\theta_s) = \text{var}(\theta_b) = \text{var}(\theta)$ and $\mathbb{E}[\theta_s] = \mathbb{E}[\theta_b] = 1$. The argument for the more general case is identical to the one below except that the parametric assumption will be different.

Assumption 4. *Assume that*

$$var(\theta) \geqslant \lambda \geqslant \left(\overline{\theta} - \underline{\theta}\right).$$

We can now prove a generalization of Proposition 7.

Proposition 15. *Under Assumption 4 and $\gamma = 1$, there exists a threshold μ_2^I such that, if $\mathbb{E}\left[\hat{\phi}_i\right]/\overline{\hat{\phi}}_i = \mathbb{E}\left[\hat{\phi}_j\right]/\overline{\hat{\phi}}_j \leqslant \mu_2^I$, there exists a unique equilibrium in which $\tilde{B}_{ii} = \tilde{B}_{ji} = \tilde{B}_{ij} = \tilde{B}_{jj} = 0$. Furthermore, $\mu_2^I > \mu_2$.*

Proof. The proof is symmetric to that of Proposition 7. First, we show the existence of an equilibrium with $\tilde{B}_{ii} = \tilde{B}_{ji} = 0$, $\tilde{B}_{ij} = \tilde{B}_{jj} = 0$ and $\tilde{B}_{if} = \tilde{B}_{jf} = y/\overline{\phi}_f$. Second, we show this equilibrium is unique.

In order for the above allocation to be part of an equilibrium, the marginal value of signing the contract in currency f has to be larger than the marginal values of doing it in currency i and j:

$$\frac{\mathbb{E}\left[\left(\theta_{js}(1+\lambda) - \theta_{ib}\right)\phi_f\right]}{\overline{\phi}_f} > \frac{\mathbb{E}\left[\left(\theta_{js}(1+\lambda) - \theta_{ib}\right)\phi_i\right]}{\overline{\phi}_i} \tag{33}$$

and

$$\frac{\mathbb{E}\left[\left(\theta_{js}(1+\lambda) - \theta_{ib}\right)\phi_f\right]}{\overline{\phi}_f} > \frac{\mathbb{E}\left[\left(\theta_{js}(1+\lambda) - \theta_{ib}\right)\phi_j\right]}{\overline{\phi}_j}. \tag{34}$$

These conditions ensure that contracts between buyers from country i and sellers from country j are set in currency f. We also need conditions for which contracts between buyers from country j and sellers of country i are set in currency f, but these are identical to the ones in the proof of Proposition 7. After substituting in the governments' best responses and evaluating these expressions at $\tilde{B}_{ii} = \tilde{B}_{ji} = 0$, $\tilde{B}_{ij} = \tilde{B}_{jj} = 0$ and $\tilde{B}_{if} = \tilde{B}_{jf} = y/\overline{\phi}_f$, these optimality conditions simplify to $\mu_1 = \mathbb{E}[\phi_f]/\overline{\phi}_f > \mathbb{E}[\hat{\phi}_i]/\overline{\hat{\phi}}_i = \mathbb{E}[\hat{\phi}_j]/\overline{\hat{\phi}}_j$. These are identical to the conditions obtained in the baseline model.

Now we show the conditions under which this equilibrium is unique. For this to be a unique equilibrium, it must also be true that the previous inequalities hold for prices ϕ_i consistent with all possible $\tilde{B}_{ii}, \tilde{B}_{ji} \in \left[0, y/\overline{\phi}_i^*\right]$. The optimal choice of inflation for the government of country i is given by

$$\phi_i = \hat{\phi}_i + \frac{1}{\psi}\left(\theta_{is}\tilde{B}_{ji} - \theta_{ib}\tilde{B}_{ii}\right).$$

Additionally, the minimum level of inflation (maximum level of ϕ) is the same as in the baseline economy: $\overline{\phi}_i = \overline{\hat{\phi}}_i + \frac{1}{\psi}\left(\bar{\theta}\tilde{B}_{ji} - \underline{\theta}\tilde{B}_{ii}\right)$. We obtain symmetric expressions for ϕ_j. Replacing the government's choice of inflation in inequality (33) yields

$$\frac{\mathbb{E}\left[\left(\theta_{js}(1+\lambda) - \theta_{ib}\right)\phi_f\right]}{\overline{\phi}_f} > \frac{\mathbb{E}\left[\left(\theta_{js}(1+\lambda) - \theta_{ib}\right)\left(\hat{\phi}_i + \frac{1}{\psi}\left(\theta_{is}\tilde{B}_{ji} - \theta_{ib}\tilde{B}_{ii}\right)\right)\right]}{\overline{\hat{\phi}}_i + \frac{1}{\psi}\left(\bar{\theta}\tilde{B}_{ji} - \underline{\theta}\tilde{B}_{ii}\right)}$$

or equivalently

$$\overline{\hat{\phi}}_i\left(\frac{\mathbb{E}[\hat{\phi}_i]}{\overline{\hat{\phi}}_i} - \frac{\mathbb{E}[\phi_f]}{\overline{\phi}_f}\right) + \frac{1}{\psi}\left(\left[1 - \frac{\mathbb{E}[\phi_f]}{\overline{\phi}_f}\bar{\theta}\right]\tilde{B}_{ji} + \left[\frac{\mathbb{E}[\phi_f]}{\overline{\phi}_f}\underline{\theta} + \frac{\text{var}(\theta)}{\lambda} - 1\right]\tilde{B}_{ii}\right) < 0. \quad (35)$$

Similarly, replacing the government's choice of inflation in inequality (34) yields

$$\frac{\mathbb{E}\left[\left(\theta_{js}(1+\lambda) - \theta_{ib}\right)\phi_f\right]}{\overline{\phi}_f} > \frac{\mathbb{E}\left[\left(\theta_{js}(1+\lambda) - \theta_{ib}\right)\left(\hat{\phi}_j + \frac{1}{\psi}\left(\theta_{js}\tilde{B}_{ij} - \theta_{jb}\tilde{B}_{jj}\right)\right)\right]}{\overline{\hat{\phi}}_j + \frac{1}{\psi}\left(\bar{\theta}\tilde{B}_{ij} - \underline{\theta}\tilde{B}_{jj}\right)}$$

or equivalently

$$\overline{\hat{\phi}}_j\left(\frac{\mathbb{E}[\hat{\phi}_j]}{\overline{\hat{\phi}}_j} - \frac{\mathbb{E}[\phi_f]}{\overline{\phi}_f}\right) + \frac{1}{\psi}\left[\left(\frac{(1+\lambda)}{\lambda}\text{var}(\theta) + 1 - \frac{\mathbb{E}[\phi_f]}{\overline{\phi}_f}\bar{\theta}\right)\tilde{B}_{ij} + \left(\frac{\mathbb{E}[\phi_f]}{\overline{\phi}_f}\underline{\theta} - 1\right)\tilde{B}_{jj}\right] < 0. \quad (36)$$

Inequalities (35) and (36) should hold for any feasible $\mathbf{B} \equiv \left\{\tilde{B}_{ii}, \tilde{B}_{ij}, \tilde{B}_{ji}, \tilde{B}_{jj}\right\}$. Since both inequalities are linear in $\mathbf{B}$, it suffices to show that they hold for all combinations of extreme values. The extreme values are computed by solving a non-linear equation for the maximum values of ϕ_i and ϕ_j. We start with inequality (35). We first check the case in which $\tilde{B}_{ji} = 0$ and $\tilde{B}_{ii} = y/\overline{\phi}_1^*$. Here $\overline{\phi}_1^*$ solves $\phi_{I1}^* = \overline{\hat{\phi}}_i - \frac{1}{\psi}\underline{\theta}y/\phi_{I1}^*$. We take the largest root of this equation which is given by $\overline{\phi}_1^* = \left(\overline{\hat{\phi}}_i + \sqrt{\left(\overline{\hat{\phi}}_i\right)^2 - 4\frac{1}{\psi}\underline{\theta}y}\right)/2$. Substituting these values in (35) yields the following inequality

$$\overline{\hat{\phi}}_i\left(\frac{\mathbb{E}\left[\hat{\phi}_i\right]}{\overline{\hat{\phi}}_i} - \frac{\mathbb{E}\left[\phi_f\right]}{\overline{\phi}_f}\right) + \frac{1}{\psi}\left[\frac{\mathbb{E}\left[\phi_f\right]}{\overline{\phi}_f}\underline{\theta} + \frac{\text{var}\left(\theta\right)}{\lambda} - 1\right]\frac{y}{\overline{\phi}_1^*} < 0. \tag{37}$$

Second, we check the other case in which $\tilde{B}_{ji} = y/\overline{\phi}_2^*$ and $\tilde{B}_{ii} = 0$. Here $\overline{\phi}_2^*$ is the largest root that solves $\phi_{I2}^* = \overline{\hat{\phi}}_i + \frac{1}{\psi}\bar{\theta}y/\phi_{I2}^*$, which is given by $\overline{\phi}_2^* = \left(\overline{\hat{\phi}}_i + \sqrt{\left(\overline{\hat{\phi}}_i\right)^2 + 4\frac{1}{\psi}\bar{\theta}y}\right)/2$. Substituting these values in in (35) yields the following inequality

$$\overline{\hat{\phi}}_i\left(\frac{\mathbb{E}\left[\hat{\phi}_i\right]}{\overline{\hat{\phi}}_i} - \frac{\mathbb{E}\left[\phi_f\right]}{\overline{\phi}_f}\right) + \frac{1}{\psi}\left[1 - \frac{\mathbb{E}\left[\phi_f\right]}{\overline{\phi}_f}\bar{\theta}\right]\frac{y}{\overline{\phi}_2^*} < 0. \tag{38}$$

Finally, we also check the case in which both $\tilde{B}_{ji}, \tilde{B}_{ii}$ are at their maximum values. In this case $\tilde{B}_{ji} = \tilde{B}_{ii} = \frac{y}{\overline{\phi}_l^*}$,where $\overline{\phi}_l^*$ is defined as in the baseline model. Substituting these values in in (35) yields the following inequality

$$\overline{\hat{\phi}}_i\left(\frac{\mathbb{E}\left[\hat{\phi}_i\right]}{\overline{\hat{\phi}}_i} - \frac{\mathbb{E}\left[\phi_f\right]}{\overline{\phi}_f}\right) + \frac{1}{\psi}\left(\frac{\text{var}\left(\theta\right)}{\lambda} - \frac{\mathbb{E}\left[\phi_f\right]}{\overline{\phi}_f}\left(\bar{\theta} - \underline{\theta}\right)\right)\frac{y}{\overline{\phi}_l^*} < 0 \tag{39}$$

We follow a symmetric approach with inequality (36). We first check the case in which $\tilde{B}_{ij} = 0$ and $\tilde{B}_{jj} = y/\overline{\phi}_1^*$. Substituting these values in (36) yields the following inequality

$$\overline{\hat{\phi}}_j\left(\frac{\mathbb{E}\left[\hat{\phi}_j\right]}{\overline{\hat{\phi}}_j} - \frac{\mathbb{E}\left[\phi_f\right]}{\overline{\phi}_f}\right) + \frac{1}{\psi}\left(\frac{\mathbb{E}\left[\phi_f\right]}{\overline{\phi}_f}\underline{\theta} - 1\right)\frac{y}{\overline{\phi}_1^*} < 0. \tag{40}$$

Second we check the other case in which $\tilde{B}_{ij} = \frac{y}{\overline{\phi}_2^*}$ and $\tilde{B}_{jj} = 0$. Substituting these values into (36) yields the following inequality

$$\overline{\hat{\phi}}_j\left(\frac{\mathbb{E}\left[\hat{\phi}_j\right]}{\overline{\hat{\phi}}_j} - \frac{\mathbb{E}\left[\phi_f\right]}{\overline{\phi}_f}\right) + \frac{1}{\psi}\left(\frac{(1+\lambda)}{\lambda}\text{var}\left(\theta\right) + 1 - \frac{\mathbb{E}\left[\phi_f\right]}{\overline{\phi}_f}\bar{\theta}\right)\frac{y}{\overline{\phi}_2^*} < 0. \tag{41}$$

Finally we also check the case in which both $\tilde{B}_{jj}, \tilde{B}_{ij}$ are at their maximum values. In this case $\tilde{B}_{jj} = \tilde{B}_{ij} = y/\overline{\phi}_l^*$,where $\overline{\phi}_l^*$ is defined as in the baseline model. Substituting these values in (36) yields the following inequality

$$\overline{\hat{\phi}}_j \left(\frac{\mathbb{E}\left[\hat{\phi}_j\right]}{\overline{\hat{\phi}}_j} - \frac{\mathbb{E}\left[\phi_f\right]}{\overline{\phi}_f} \right) + \frac{1}{\psi} \left(\frac{(1+\lambda)}{\lambda} \mathrm{var}\left(\theta\right) - \frac{\mathbb{E}\left[\phi_f\right]}{\overline{\phi}_f} \left(\bar{\theta} - \underline{\theta}\right) \right) \frac{y}{\overline{\phi}_l^*} < 0. \tag{42}$$

Now we need to show that inequalities (37) - (42) are satisfied for values of policy risk such that $\mathbb{E}\left[\hat{\phi}_i\right] / \overline{\hat{\phi}}_i = \mathbb{E}\left[\hat{\phi}_j\right] / \overline{\hat{\phi}}_j \leqslant \mu_2$. First note that (40) always holds since the second term is negative. Additionally, if (41) holds then (38) is also satisfied. Finally, if (42) holds then (39) is also satisfied. This leaves us with (37), (41) and (42). It is worth noting that $\overline{\phi}_2^* > \overline{\phi}_l^* > \overline{\phi}_1^*$. Also recall that $\mathbb{E}\left[\hat{\phi}_i\right] / \overline{\hat{\phi}}_i = \mathbb{E}\left[\hat{\phi}_j\right] / \overline{\hat{\phi}}_j \leqslant \mu_2$ implies that

$$\overline{\hat{\phi}}_j \left(\frac{\mathbb{E}\left[\hat{\phi}_j\right]}{\overline{\hat{\phi}}_j} - \frac{\mathbb{E}\left[\phi_f\right]}{\overline{\phi}_f} \right) + \frac{1}{\psi} \left(\frac{(2+\lambda)}{\lambda} \mathrm{var}\left(\theta\right) - \frac{\mathbb{E}\left[\phi_f\right]}{\overline{\phi}_f} \left(\bar{\theta} - \underline{\theta}\right) \right) \frac{y}{\overline{\phi}_l^*} < 0. \tag{43}$$

It then follows that if $\mathbb{E}\left[\hat{\phi}_i\right] / \overline{\hat{\phi}}_i = \mathbb{E}\left[\hat{\phi}_j\right] / \overline{\hat{\phi}}_j \leqslant \mu_2$ (or equivalently if (43) holds), then (42) is satisfied. Additionally, note that if we use the assumption that $\mathrm{var}(\theta) > \lambda$ then (43) implies (41). Finally, we show that (43) implies (37). To show this we must have that

$$\left(\overline{\hat{\phi}}_i + \sqrt{\overline{\hat{\phi}}_i^2 - 4\frac{1}{\psi}\underline{\theta} y} \right) \left(\frac{(2+\lambda)}{\lambda} \mathrm{var}\left(\theta\right) - \frac{\mathbb{E}\left[\phi_f\right]}{\overline{\phi}_f} \left(\overline{\theta} - \underline{\theta}\right) \right) >$$
$$\left(\overline{\hat{\phi}}_i + \sqrt{\overline{\hat{\phi}}_i^2 + 4\frac{1}{\psi}\left(\overline{\theta} - \underline{\theta}\right) y} \right) \left(\frac{\mathrm{var}\left(\theta\right) - \lambda}{\lambda} + \underline{\theta}\frac{\mathbb{E}\left(\phi_f\right)}{\overline{\phi}_f} \right).$$

This can be rewritten as

$$\begin{aligned} &\frac{\mathrm{var}\left(\theta\right)}{\lambda} \left[\left(\overline{\hat{\phi}}_i + \sqrt{\overline{\hat{\phi}}_i^2 - 4\frac{1}{\psi}\underline{\theta} y} \right) (2+\lambda) - \left(\overline{\hat{\phi}}_i + \sqrt{\overline{\hat{\phi}}_i^2 + 4\frac{1}{\psi}\left(\overline{\theta} - \underline{\theta}\right) y} \right) \right] \\ &- \frac{\mathbb{E}\left[\phi_f\right]}{\overline{\phi}_f} \left[\left(\overline{\hat{\phi}}_i + \sqrt{\overline{\hat{\phi}}_i^2 - 4\frac{1}{\psi}\underline{\theta} y} \right) \left(\overline{\theta} - \underline{\theta}\right) \right] \\ &+ \left(\overline{\hat{\phi}}_i + \sqrt{\overline{\hat{\phi}}_i^2 + 4\frac{1}{\psi}\left(\overline{\theta} - \underline{\theta}\right) y} \right) \left[1 - \frac{\mathbb{E}\left[\phi_f\right]}{\overline{\phi}_f}\underline{\theta} \right] > 0. \end{aligned} \tag{44}$$

First consider the term

$$\begin{aligned}
&\left(\overline{\hat{\phi}}_i+\sqrt{\overline{\hat{\phi}}_i^2-4\frac{1}{\psi}\underline{\theta}y}\right)(2+\lambda)-\left(\overline{\hat{\phi}}_i+\sqrt{\overline{\hat{\phi}}_i^2+4\frac{1}{\psi}\left(\overline{\theta}-\underline{\theta}\right)y}\right)\\
\geqslant&\left(\overline{\hat{\phi}}_i+\sqrt{\overline{\hat{\phi}}_i^2-4\frac{1}{\psi}\underline{\theta}y}\right)(2+\lambda)-\left(\overline{\hat{\phi}}_i+\sqrt{\overline{\hat{\phi}}_i^2-4\frac{1}{\psi}\underline{\theta}y}+\sqrt{4\frac{1}{\psi}\overline{\theta}y}\right)\\
=&(1+\lambda)\overline{\hat{\phi}}_i-\sqrt{4\frac{1}{\psi}\overline{\theta}y}+(1+\lambda)\sqrt{\overline{\hat{\phi}}_i^2-4\frac{1}{\psi}\underline{\theta}y}\\
\geqslant&0
\end{aligned}$$

if $\overline{\hat{\phi}}-\sqrt{4\frac{1}{\psi}\overline{\theta}y}\geqslant 0$, which is a condition we need for $\overline{\phi}_1^*$ to be well-defined. Given this, the first two lines of (44) are greater than

$$\frac{\mathbb{E}\left[\phi_f\right]}{\overline{\phi}_f}\left(\left(\lambda-\left(\overline{\theta}-\underline{\theta}\right)\right)\overline{\hat{\phi}}+\overline{\hat{\phi}}-\sqrt{4\frac{1}{\psi}\overline{\theta}y}+\left(1+\lambda-\left(\overline{\theta}-\underline{\theta}\right)\right)\sqrt{\overline{\hat{\phi}}^2-4\frac{1}{\psi}\underline{\theta}y}\right),$$

which is positive if $\lambda\geqslant\left(\overline{\theta}-\underline{\theta}\right)$. Hence, we showed that (37) - (42) are satisfied for values of policy risk such that $\mathbb{E}\left[\hat{\phi}_i\right]/\overline{\hat{\phi}}_i=\mathbb{E}\left[\hat{\phi}_j\right]/\overline{\hat{\phi}}_j\leqslant\mu_2$. Finally, the cutoff value μ_2^I is defined as the smallest cutoff value such that (37) - (42) are satisfied. Q.E.D.

C.7 Model of a Credit Chain

We now present a simple credit chain model that endogenizes the stocks of foreign and local currency in Section 3.3.

Suppose that citizens are further divided into one of I sub-types $\mathcal{I}\in\{1,2,...,I\}$ with a continuum of each. A citizen of type i has preferences over a special good produced by type $i+1$ and produces a special good valued by type $i-1$. All types also value the consumption of the numeraire good, which takes place at the end of period 2. Preferences for the representative citizen type i are given by

$$u_i=(1+\lambda)\,{}_ix_{i+1}-{}_{i-1}x_i+\mathbb{E}\left[\theta_i c_i\right],$$

where ${}_ix_{i+1}$ is the special good produced by a citizen of type $i+1$ for a citizen of type i and ${}_{i-1}x_i$ is the special good produced by a citizen of type i for a citizen of type $i-1$. We assume that ${}_0x_1={}_Ix_{I+1}=0$ so that type 1 does not produce a special good for any other type and type I does not consume a special good. We assume that $\theta_i\in\left[\underline{\theta},\overline{\theta}\right]$ is independent across sub-types and that $\mathbb{E}\left[\theta_i\right]=1$.

The timing of the model is as follows:

1. The first period $t = 1$ is divided into $I - 1$ sub-periods in which trade takes place sequentially:

   (a) In sub-period 1, citizens of type 2 produce a special good for citizens of type 1 in exchange for the promise of payment in period 2.

   (b) Similarly, in sub-period i, citizens of type $i + 1$ produce a special good for citizens of type i in exchange for the promise of payment in period 2.

2. The second period $t = 2$ is divided into three sub-periods:

   (a) In sub-period 1, the taste shocks θ_i are realized for all citizens.

   (b) In sub-period 2, the government chooses its policy, which is the aggregate price level.

   (c) In sub-period 3, all signed contracts are executed in the order in which they were signed and, finally, consumption of the composite good takes place.

Assume that all citizens are endowed with y units of the numeraire good. The definition of a bilateral contract between i and $i + 1$ is identical to Section 3.3. Note that in this contract $i + 1$ is the seller and i is the buyer. Given the structure of the credit chain, $(\hat{b}_f, \hat{b}_l)$ is the promised payment to type i from types $i - 1$.

We can then use Propositions 2 and 9 to characterize the bilateral contract.

Proposition 16. *In the optimal bilateral contract, the amount of special good is given by* $x = \mathbb{E}\left[\theta_s\left(b_l\phi_l + b_f\phi_f\right)\right]$, *while the payments satisfy*

1. *If* $\mathbb{E}\left[\left(\theta_s(1+\lambda) - \theta_b\right)\frac{\phi_l}{\bar{\phi}_l}\right] < \mathbb{E}\left[\left(\theta_s(1+\lambda) - \theta_b\right)\frac{\phi_f}{\bar{\phi}_f}\right]$, *then* $b_l = \hat{b}_l$ *and* $b_f = \hat{b}_f + \frac{y}{\bar{\phi}_f}$.

2. *If* $\mathbb{E}\left[\left(\theta_s(1+\lambda) - \theta_b\right)\frac{\phi_l}{\bar{\phi}_l}\right] = \mathbb{E}\left[\left(\theta_s(1+\lambda) - \theta_b\right)\frac{\phi_f}{\bar{\phi}_f}\right]$, *then* $b_l = \hat{b}_l + \rho\frac{y}{\bar{\phi}_l}$ *and* $b_f = \hat{b}_f + (1-\rho)\frac{y}{\bar{\phi}_f}$ *for any* $\rho \in [0, 1]$.

3. *If* $\mathbb{E}\left[\left(\theta_s(1+\lambda) - \theta_b\right)\frac{\phi_l}{\bar{\phi}_l}\right] > \mathbb{E}\left[\left(\theta_s(1+\lambda) - \theta_b\right)\frac{\phi_f}{\bar{\phi}_f}\right]$, *then* $b_l = \hat{b}_l + \frac{y}{\bar{\phi}_l}$ *and* $b_f = \hat{b}_f$.

The result follows immediately from Propositions 2 and 9. In particular, the optimal contract will feature currency matching of stocks and will denominate the flows in the currency with the largest marginal benefit.

D Figures

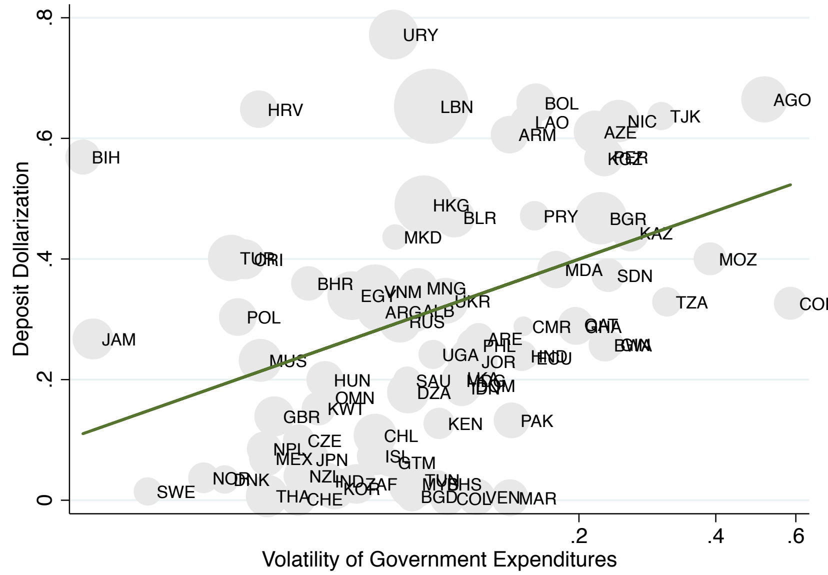

Figure D.1: Financial Dollarization and Fiscal Policy Risk

Notes: Deposit dollarization is measured as the share of bank deposits denominated in US dollars. The source of this data is Levy-Yeyati (2006). The horizontal axis (in log scale) shows the volatility of government expenditures across countries. The source of this data is the World Bank. Volatility is computed as the standard deviation of the cyclical component of real government expenditures. The size of each point corresponds to the average seigniorage collected in that country, measured as the change in monetary aggregates as a fraction of GDP. The source of this data is the World Bank.

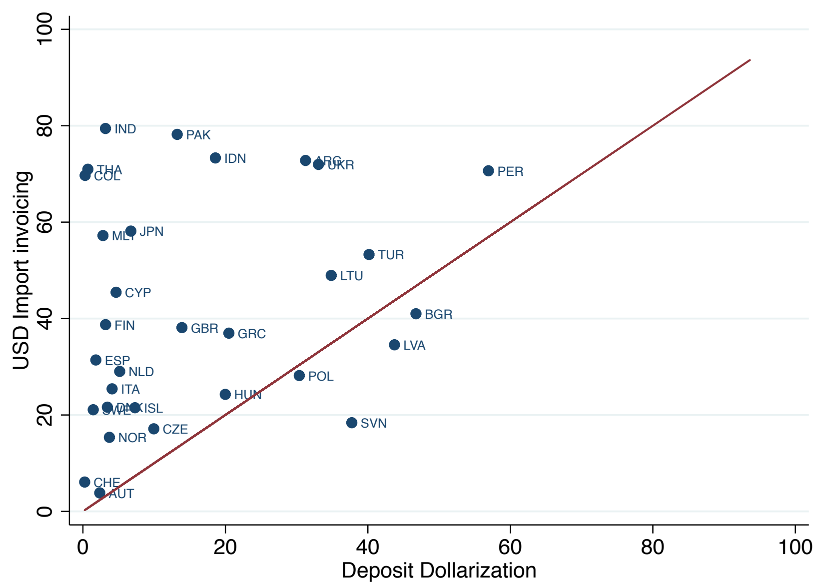

Figure D.2: Financial versus International Trade Dollarization

Notes: Financial dollarization is measured as the share of bank deposits denominated in US dollars. The source of this data is Levy-Yeyati (2006). Inflation volatility is measured as the standard deviation of annual inflation for the period 1980-2017. The source of this data is IFS. Trade Dollarization is computed as the share of imports, from destinations other than the US, invoiced in US dollars. The source of this data is Gopinath (2016).

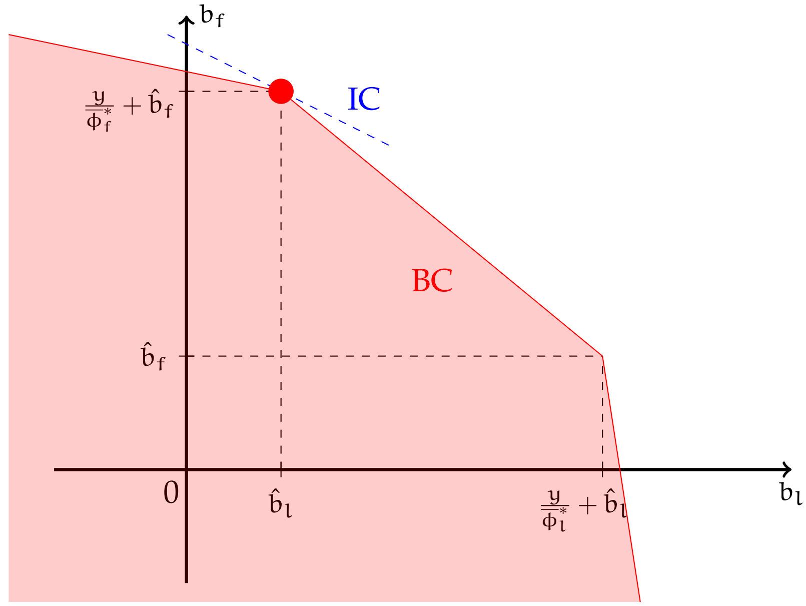

Figure D.3: Optimal Bilateral Contract with Outstanding Currency Claims

Notes: This figure depicts the solution of the individual contract problem for an illustrative case. The set BC refers to the set of promised payments that satisfy the payments feasibility constraint. This set has three regions:

$$\begin{aligned}
&\overline{\phi}_l b_l + \overline{\phi}_f b_f \leqslant y \quad \text{if } b_l \geqslant \hat{b}_l \text{ and } b_f \geqslant \hat{b}_f, \\
&\overline{\phi}_l b_l + \underline{\phi}_f b_f \leqslant y \quad \text{if } b_l \geqslant \hat{b}_l \text{ and } b_f < \hat{b}_f, \\
&\underline{\phi}_l b_l + \overline{\phi}_f b_f \leqslant y \quad \text{if } b_l < \hat{b}_l \text{ and } b_f \geqslant \hat{b}_f.
\end{aligned}$$

The IC line corresponds to the indifference curve $\bar{U} = \mathbb{E}\left([(1+\lambda)\theta_s - \theta_b](\phi_l b_l + \phi_f b_f)\right)$. In this illustration the optimal contract is given by the red point. More generally, Assumption 3 guarantees that any optimal contract will feature $b_l > \hat{b}_l$ and $b_f > \hat{b}_f$